Enhanced interleaved partitioning PTS for peak-to-average power ratio reduction in OFDM systems

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The independence of the candidates generated in the existing interleaved partitioning partial transmit sequences (IP-PTS) is first investigated. The result shows that these candidates are not fully independent, which leads to performance inferior to the adjacent partitioning PTS (AP-PTS) in which all the generated candidates are mutually independent. To improve the performance of the IP-PTS in peak-to-average power ratio reduction, a novel type of phase sequences for IP-PTS is proposed that can be used to produce fully independent candidates so that IP-PTS can achieve similar performance to AP-PTS, while keeping low complexity.

Introduction: Partial transmit sequences (PTS) [1, 2] is one of the promising methods in reducing the peak-to-average power ratio (PAPR) of OFDM signals. PTS is usually implemented in such a way that the original frequency domain (FD) data sequence is first divided into disjoint subblocks using a certain partitioning scheme, e.g. adjacent partitioning, interleaved partitioning or pseudorandom partitioning [3], then the subblocks are multiplied with phase sequences to create a set of candidates, and finally one candidate with the lowest PAPR is chosen from the candidates for transmission. The performance of PTS in PAPR reduction is related to the number of generated candidates and the partitioning scheme.

Interleaved partitioning partial transmit sequence (IP-PTS) has been shown to have low computational complexity [3, 4] but inferior performance compared to adjacent partitioning PTS (AP-PTS) and pseudorandom partitioning PTS (PRP-PTS) when the number of generated candidates are the same [3, 5]. The question 'Why does IP-PTS have inferior performance?' motivated the present work. An extensive literature study has been conducted by the authors but no answer to this question has been found. After this study, an investigation of the independence of candidates generated in IP-PTS has been carried out, the answer to the question is found, and a new type of phase sequences is proposed that can create fully independent candidates. Simulation studies have been conducted and the results show that for the same number of candidates the performance of IP-PTS using new phase sequences can be improved to achieve similar performance to AP-PTS. IP-PTS with enhanced performance due to using such new phase sequences is called enhanced IP-PTS (EIP-PTS).

Background of IP-PTS: Conventionally in a PTS method an FD data, $\boldsymbol{X} = \{X_k\}$ $(k = 0, \ldots, N-1)$, of an OFDM system with N subcarriers is divided by means of a certain partitioning scheme into V disjoint subblocks $\boldsymbol{X}^{(v)}$ so that $\boldsymbol{X} = \sum_{v=0}^{V-1} \boldsymbol{X}^{(v)}$ and then the M candidates are generated in the following manner

$$x_n^\zeta = IDFT\left\{\sum_{v=0}^{V-1} b_v^\zeta X_k^{(v)}\right\} = \sum_{v=0}^{V-1} b_v^\zeta x_n^{(v)}, \quad \zeta = 1, \ldots, M \qquad (1)$$

where $x_n^{(v)} = IDFT\{X_k^{(v)}\}$ are the so-called partial transmit sequences (PTSs), and b_v^ζ are the weighting (or phase-rotation) factors. b_v^ζ are often created by setting $b_0^\zeta = 1$(without loss of generality) and choosing b_v^ζ, $v = 1, \ldots, V-1$, from four values $\{\pm j, \pm 1\}$. In this case, one may have $M = 4^{V-1}$ candidates x_n^ζ $(\zeta = 1, \ldots, M)$, from which the candidate with the smallest PAPR is selected for transmission.

An alternative general expression for (1) is of the following form [4]

$$\begin{aligned} \boldsymbol{x}^\zeta &= IDFT\{\boldsymbol{X}^\zeta\} = IDFT\{\boldsymbol{P}^\zeta \boldsymbol{X}\} \\ &= IDFT\{[P_0^\zeta X_0, P_1^\zeta X_1, \ldots, P_{N-1}^\zeta X_{N-1}]\} \end{aligned} \qquad (2)$$

where $\boldsymbol{P}^\zeta = \{P_k^\zeta\}$ are phase sequences that have different structures for different portioning schemes.

For IP-PTS, the phase sequences $\boldsymbol{P}^\zeta$ have a periodic structure with a period of V, expressed as

$$\boldsymbol{P}^\zeta = [\underbrace{\boldsymbol{B}^\zeta, \boldsymbol{B}^\zeta, \ldots, \boldsymbol{B}^\zeta}_{N/V = L}] \qquad (3)$$

where

$$\boldsymbol{B}^\zeta = [b_0^\zeta, b_1^\zeta, \ldots, b_{V-1}^\zeta] \qquad (4)$$

is a phase vector of length V. And V, the number of subblocks, is chosen to be such a number that $L = N/V$ is also an integer, and $P_{v+lV}^\zeta = b_v^\zeta$ $(v = 0, \ldots, V-1, l = 0, \ldots, L-1)$

Independence of candidates: The performance of a PTS in PAPR reduction is affected by the number of independent (or effective) candidates. Each of the independent candidates has a different PAPR from the others. Therefore, the more independent candidates, the better the performance. The fact that IP-PTS has been shown to be inferior in performance to AP-PTS for the same number of candidates should, thus, be related to the independence of candidates. To study the independence of candidates generated in IP-PTS, we first introduce the following lemma.

Lemma: If a phase sequence $\tilde{\boldsymbol{P}} = \{\tilde{\boldsymbol{B}}, \tilde{\boldsymbol{B}}, \ldots, \tilde{\boldsymbol{B}}\}$ with L $\tilde{\boldsymbol{B}}$s, where $\tilde{\boldsymbol{B}} = [\tilde{\boldsymbol{b}}_0, \tilde{\boldsymbol{b}}_1, \ldots, \tilde{\boldsymbol{b}}_{V-1}]$ is constructed from the relation $\tilde{\boldsymbol{B}}_v = W_V^{-vl} B_v$ $(l = \pm 1, \ldots, \pm(V-1), v = 0, \ldots, V-1)$, then $\tilde{x}_n = IDFT\{\tilde{P}_k X_k\}$ is the circular shift of sequence $\bar{x}_n = IDFT\{P_k X_k\}$, i.e.

$$\tilde{x}_n = \bar{x}_{n+lL} \qquad (5)$$

where $W_V^{-vl} = \exp(j(2\pi/V)vl)$ is the twiddle factor and $\boldsymbol{P} = \{\boldsymbol{B}, \boldsymbol{B}, \ldots, \boldsymbol{B}\}$ has the same form as (3). The lemma can be easily be proven by noticing that $\tilde{P}_k = W_N^{-lLk} P_k$.

Obviously, $\tilde{x}_n$ and $\bar{x}_n$ have the same PAPR, and they are not independent. Thus, in the existing IP-PTS, the number of effective candidates is less than the total number of candidates. This may explain the reason that IP-PTS has inferior performance to AP-PTS. For example, for IP-PTS with $V = 4$, the candidates generated from four phase vectors, $\{1, 1, 1, 1\} = \{B_k\}$, $\{1, j, -1, -j\} = \{W_4^{-k} B_k\}$, $\{1, -1, 1, -1\} = W_4^{-2k} B_k$ and $\{1, -j, -1, j\} = \{W_4^{-3k} B_k\}$, have the same PAPR according to the lemma. It can be demonstrated that the number of effective candidates in IP-PTS is 2 and 16, respectively, for $V = 2$ and 4, though the number of the total candidates is 4 and 64 candidates, respectively.

Enhanced IP-PTS (EIP-PTS): It is shown that a less number of effective candidates generated in IP-PTS leads to inferior performance to AP-PTS. Although dividing the FD data into more subblocks may result in the increased number of effective candidates, this inevitably increases the amount of side information (SI) and the computational costs. Here, a new type of the phase sequences is proposed that, unlike (3), are of form

$$\boldsymbol{P}^\zeta = [\underbrace{\boldsymbol{D}^\zeta, \boldsymbol{D}^\zeta, \ldots, \boldsymbol{D}^\zeta}_{N/(2V)}] \qquad (6)$$

where $\boldsymbol{D}^\zeta \triangleq [d_0^\zeta, d_1^\zeta, \ldots, d_{V'-1}^\zeta] = [b_0^\zeta, b_1^\zeta, \ldots, b_{V-1}^\zeta, b_{V-1}^\zeta, \ldots, b_1^\zeta, b_0^\zeta]$, $V' = 2V$, and b_v^ζ $(v = 0, \ldots, V-1)$ are the same as those in (4). The phase sequences in (6) can be seen as if the FD data was divided into V' subblocks, but only V phase elements need to be optimised (so that the SI is not increased). It can be verified via computer simulations that all the $M = 4^{V-1}$ candidates generated in such a way are independent.

Owing to the periodic structure of phase sequences in (6), the computational costs of the enhanced IP-PTS (EIP-PTS) can remain low using the methods in both [3] and [4]. For example, from [4], the candidates can be written as

$$x_n^\zeta = \sum_{\substack{i=0 \\ i \neq V'/2}}^{V'-1} q_i^\zeta \{x((n - iL'))_N R_n\}, \quad \zeta = 1, \ldots, M \qquad (7)$$

where $\boldsymbol{q}^\zeta = IDFT\{\boldsymbol{D}^\zeta\}$, $((n))_N$ denotes n modulo N, and R_n is the rectangular sequence. $i = V'/2$ is not included in the superposition in (7) because of $q_{V'}^\zeta/2 = 0$. The candidate with the smallest PAPR is selected for transmission from the resulting candidates x_n^ζ. Equations (6) and (7) constitute the EIP-PTS.

Simulations and results: In simulations, the complement cumulative distribution function (CCDF) of the PAPR of more than 100000 OFDM signals with $N = 128$, $CCDF = \Pr\{PAPR > PAPR_0\}$, is used to evaluate the performance of different PAPR reduction methods. Fig. 1 shows the CCDFs of the PAPR of QPSK-modulated OFDM signals in the EIP-PTS, AP-PTS and the existing IP-PTS, for $V = 2$ and 4, respectively. The numbers of effective candidates for the EIP-PTS, AP-PTS and the existing IP-PTS are 64, 64 and 16, respectively, for $V = 4$. Because of the less number of effective candidates in the existing IP-PTS, its performance is inferior to AP-PTS, while AP-PTS and EIP-PTS have similar performance. For instance, 0.1% PAPR of the normal OFDM is about 11.4 dB, and, for $V = 4$, 0.1% PAPR of the enhanced IP-PTS, AP-PTS and the conventional IP-PTS are about 7.8, 7.8 and 8.8 dB, respectively. AP-PTS and EIP-PTS have about 1 dB performance gain compared to the existing IP-PTS.

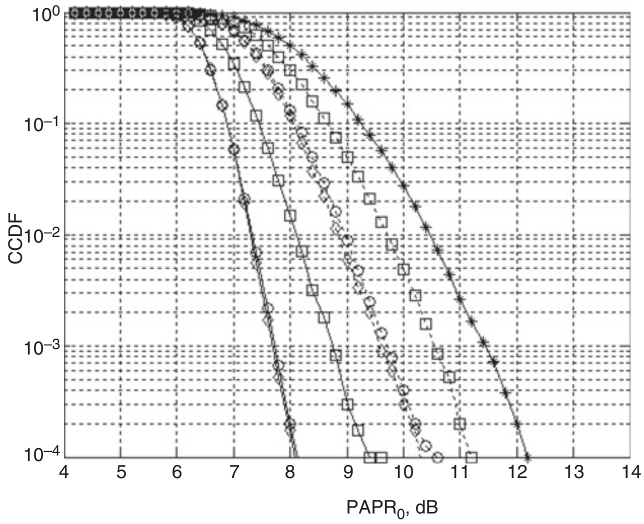

Fig. 1 *CCDFs of AP-PTS, existing IP-PTS and EIP-PTS*

—∗— normal OFDM
– – – $V = 2$
—— $V = 4$
◇ AP-PTS
□ existing IP-PTS
o EIP-PTS

Conclusions: The independence of the candidates in the existing IP-PTS has been studied. The study has shown that the candidates generated in the existing IP-PTS are not fully independent, which results in the less number of effective candidates and, thus, inferior performance compared to AP-PTS. A new type of phase sequences has been proposed that can create fully independent candidates to enhance the performance of the existing IP-PTS. Simulations have been conducted and the results show that the EIP-PTS using these new phase sequences can be improved to a similar performance to that of AP-PTS, while keeping low complexity.

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