

# Novel Method for Simulation of Lamb Wave Propagation Generated by an Interdigital Transducer

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## ABSTRACT

Recent years have brought great changes in the approach to the systems condition monitoring. Instead of using temporary attached sensors for scheduled technical inspection more and more often permanent health monitoring systems are mounted on structures. One of the techniques used for structural health monitoring systems is the Lamb waves-based technique. In this paper a novel method for modeling and simulation of complex shape transducers, used for the Lamb waves excitation and sensing, is presented and illustrated on the example of Interdigital Transducer (IDT). The proposed method is computationally efficient - it allows for an approximate prediction of the wave-field generated by an arbitrary-shaped transducer in a relatively short time.

## INTRODUCTION

Non destructive testing (NDT) and structural health monitoring (SHM) systems have been constantly improved during past decades [1]. These types of systems can be based on a variety of physical phenomena. One of the methods recently introduced for the system condition evaluation is the technique based on ultrasonic surface waves propagation, especially Lamb waves (LWs). This type of waves can exist in thin plates with parallel free boundaries and may be used for the detection of structural defects in different types of materials [2]. LWs are sensitive not only for the surface flaws but can be used for the detection of multiple defects in entire cross section of the structure [3].

Among the transducers that may be used for LWs' generation, interdigital transducers (IDTs) are gaining increasing recognition.

The Interdigital transducers for use in NDT and SHM systems are relatively novel construction. The structure of a typical IDT, presented in figure 1, consists of three main layers: 1. Bottom electrode(s); 2. Piezoelectric material; 3. Top electrodes.

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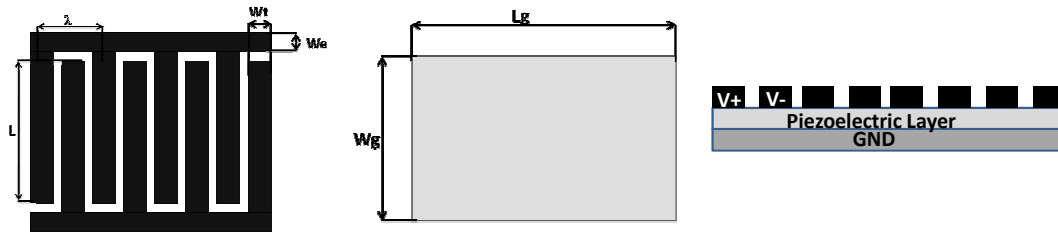


Figure.1. Structure of a single sided IDT transducer (top electrode, bottom electrode, cross section)

There are two versions of the IDTs: a single sided design where only one side of the transducer is covered by interdigital electrodes when other side is made as plain surface covered with conductive material and a double sided IDT that has comb-shaped electrodes formed on the both sides of the transducer. The distance between the opposite phase electrodes (fingers separation)[4] defines the length of the induced wave[5,6]. Other aspect of the IDT transducers is their ability for directional waves generation [7,8]. The wave beam is generated in the direction perpendicular to the finger electrodes and the divergence of the wave is dependent on the fingers length. The ultrasonic transducers, including IDTs, can be modeled using different techniques, for instance finite elements (FE)[8] or finite differences methods[9]. Although these methods allow to create accurate models of the transducers, due to high time resolution required to simulate the high frequency responses, the computational time can be significant. In some cases accurate models are not required and therefore approximate methods can be used.

The method proposed in [10] assumed that a finger of an IDT can be modeled by a set of omnidirectional wave point sources. Next, the acoustic field at a given distance was calculated and the response due to overall excitation can be found as a superposition of all emitted waves. In [11] the out-of-plane displacement at a given distance was predicted based on the Victorov plate theory using the expressions given in[12].

Here, the authors present an approximate method for the simulation of IDTs, which also assumes the introduction of a set of omnidirectional point sources and the Huygen's principle to take into account interference of the excited waves. However, a simplified model for the phenomenon of wave propagation is considered. The response at a given point is evaluated as a convolution between the excited signal and the structure's transfer function (STF).

## APPROXIMATE MODEL DEVELOPMENT

The implementation proposed in this paper, assumes a set of omnidirectional wave point sources, distributed according to the transducer geometry (figure 2).

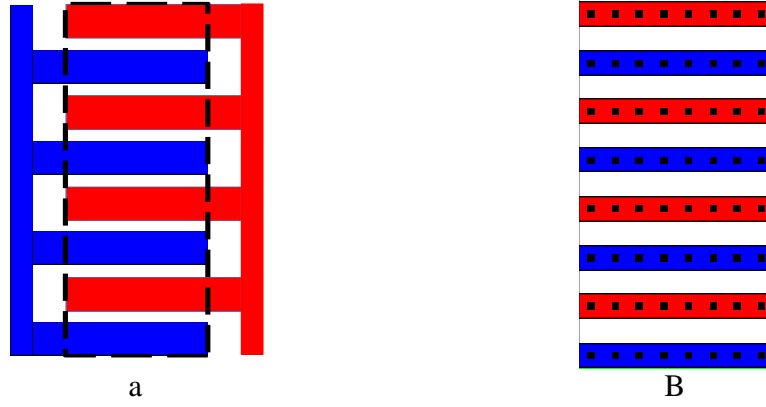


Figure 2. Layout of the electrode pattern for the IDT used in numerical simulation: a) full pattern with an active area marked (dashed line) b) an active area pattern used for approximate model development with point sources localization

Each source generates an omnidirectional ultrasonic Lamb wave, which propagates in a thin plate according to the predefined frequency-dependent structure's transfer function (STF) [13]. The response of the surface  $V_r(t)$  on dispersive Lamb waves, excited by the signal  $V_e(t)$  may be calculated using the following relation:

$$V_r(t) = F^{-1}(V_e(\omega)G(k, \omega, x)) \quad (1)$$

where  $G(k, \omega, x)$  is the frequency dependent STF,  $k$  is the wavenumber of the propagating mode,  $x$  is the propagation distance and  $F^{-1}$  stands for inverse Fourier transform. In the case when multiple symmetric (S) and antisymmetric (A) modes are present, the function  $G(k, \omega, x)$  takes the form:

$$G(k, \omega, x) = \sum_{j=1}^n S_j(\omega)e^{ik^S_j x} + \sum_{l=1}^m A_l(\omega)e^{ik^{A_l} x} \quad (2)$$

where  $S_j$  and  $A_l$  correspond to the amplitudes of the received S and A modes, respectively.

Next, in order to model the wave-field generated by the transducer, the Huygens' principle is adapted. According to this principle [10], the signal evaluated at individual point is a linear superposition of waves generated by the point sources (figure 2b) and can be written as:

$$V_{IDT}(t) = \sum_{g=1}^w P^g(t)V_r^g(t) \quad (3)$$

where  $V_{IDT}(t)$  is the total response of the surface due to the excitation  $V_e(t)$  generated by the transducer,  $V_r^g(t)$  is the response from a single source  $g$ ,  $P^g(t)$  is the weighting factor that allows for the correction of the amplitude separately for each electrode.

## STF SIMULATION SETUP

The proposed method was implemented in MATLAB® environment as a set of functions. The first step of the simulation procedure is the definition of the geometrical

parameters for the modeled transducer which, as presented in figure 2a is discretized into a set of point sources. Based on the work of Wilcox and own research, considered as beyond the scope of this paper, the discretization step was set as 4 point sources per wavelength and each of the electrode was modeled as a single line of point sources as presented in figure 2b.

Next, the geometry of the structure and the positions of the measurement points are defined and the matrix containing all the distances for each pair a point source – a measuring point is created. The final steps are calculations of the wave propagation. First, the point-to-point response for each distance is calculated based on the frequency-dependent transfer function. In the last step the overall response in each measuring point is obtained based on calculated time series and power supply configuration (gain, and phase for each finger electrode).

## NUMERICAL SIMULATIONS

### Influence of dispersion on IDT simulation results

To verify the results obtained with the proposed method numerical models of the IDT placed on an aluminum plate were created using the developed approximate technique. The IDT model used for the simulations was designed for the excitation of the A0 mode at frequency 330kHz in a 4mm thick aluminum plate. Based on the relationships presented in [8] the following dimensions of the transducer were calculated:  $L=15\text{mm}$ ,  $\lambda=7.5\text{mm}$ ,  $W_t=1.9\text{mm}$   $W_e=0.75\text{mm}$  (see figure 1).

The first set of simulations was focused on the identification of the influence of the dispersion on the shape of the beam-pattern. The measuring points were defined in a pattern of a circles with the radius equal 150mm and 900mm, as presented in figure 3. For tests, two excitation signals were used at two frequencies: 100kHz and 330kHz. The first frequency was in a dispersive part of the curves whereas the second was at a point with minimal dispersion.

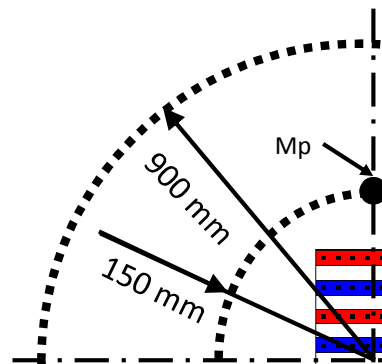


Figure 3. Location of measuring points during simulations

For the both of selected frequencies the following excitation signals were considered: first a wide-band signal consisting of 3 cycles of sine modulated with a Hanning window was applied, next the number of cycles was increased up to 30 to get the narrow-band signal and to reduce the effect of dispersion.

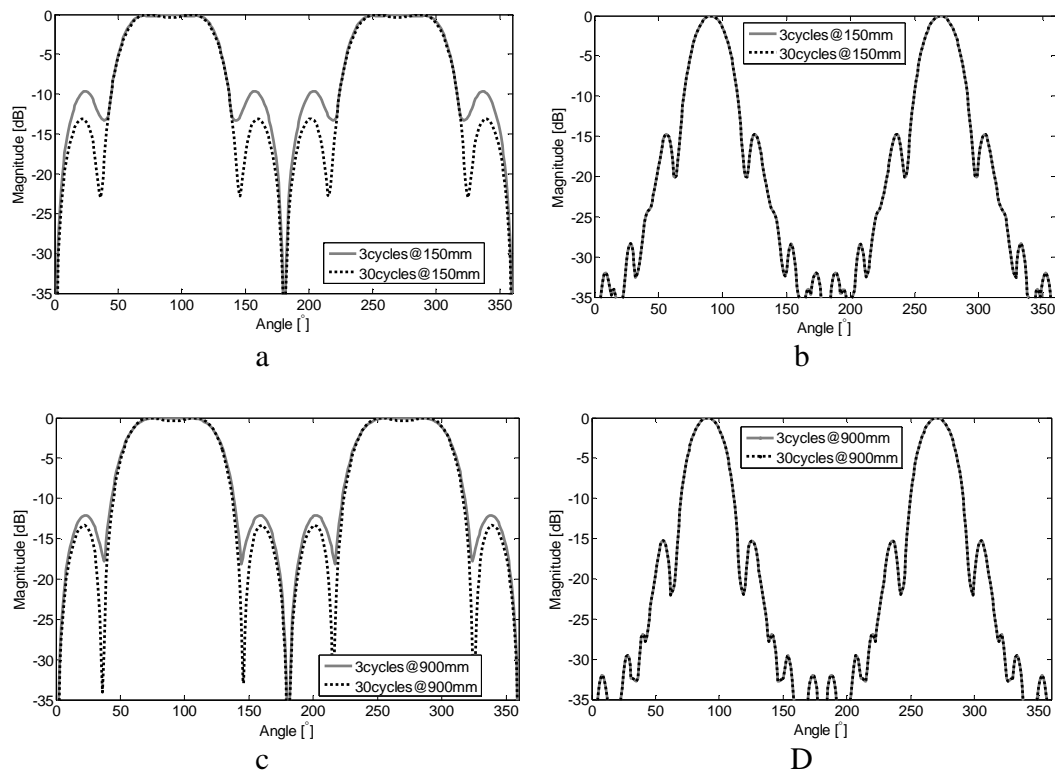


Figure 4. Beam patterns evaluated for the waves excited with windowed tone burst consisting of 3 and 30 sine cycles using STF method at: a) distance of 150mm and frequency 100kHz, b) distance of 150mm and frequency 330kHz, c) distance of 900mm and frequency 100kHz and d) distance of 900mm and frequency 330kHz

In figure 4 the beam-patterns obtained at distance of 150mm and 900mm for both frequencies and both excitation signals were presented. It is visible that for frequency 330kHz, where the dispersion of waves is minimal and the electrode spacing of the IDT is equal to the length of the generated waves (figure 4b and figure 4d), the changes in the number of cycles in excitation signals or distance from the transducer have almost no influence on the generated beam-pattern. Significant differences are however found for the excitation frequency 100kHz (figure 4a and figure 4c). For the angle intervals  $7-42^\circ$  and  $140-176^\circ (+180^\circ)$  the beam-pattern obtained with the wide-band excitation is flattened compared to the narrow-band one for both of the tested distances. Moreover, the similar effect may be observed if the beam-patterns from different distances are compared. Results obtained from distance of 150mm are flattened compared to these from 900mm in given intervals.

To verify the accuracy of the proposed method, the second set of simulations were performed for different excitation frequencies and compared to the ones obtained from commercially available FE software (ANSYS Multiphysics). The model of the structure created in ANSYS Multiphysics was built using 20-node brick finite elements. Fully coupled transient analyses were performed to simulate the piezoelectric effect present in the IDT. The simulations were performed with the same settings: the simulation time was  $80\mu\text{s}$  and the transducers were excited with the same electrical signals (a five-cycle tone burst modulated with a Hanning window with the amplitude  $100V_{\text{p-p}}$ ).

## FEM validation of the proposed technique

Because there is no direct relationship between material properties and amplitude of generated signal, moreover there is no damping in the approximate model, only a qualitative comparison could be done between the results obtained from the simulations carried for FE and STF based models.

The beam patterns, calculated for each excitation frequency using the obtained time series, were eventually taken for results comparison. For each measuring point the Hilbert transform was used to determine the envelope of the snapshots for the out-of-plane velocity. Next, the maximal value found for each envelope was used for beam pattern calculation. Finally the maximal value determined for each beam pattern for given simulation was found and then used for amplitude normalization. In figure 5 the beam patterns for 100kHz, 330kHz and 425kHz are presented. It is clear that a good agreement between the proposed model and FEM results can be observed for all tested frequencies.

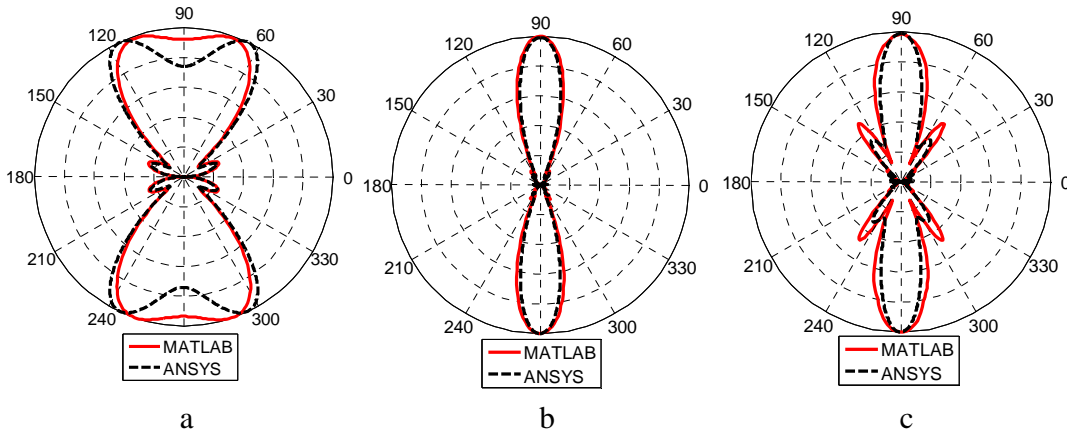


Figure 5. Normalized beam patterns of the IDT obtained during simulations with FEA (ANSYS) and proposed approximate (MATLAB) models at: a) 100kHz, b) 330kHz c) 425kHz.

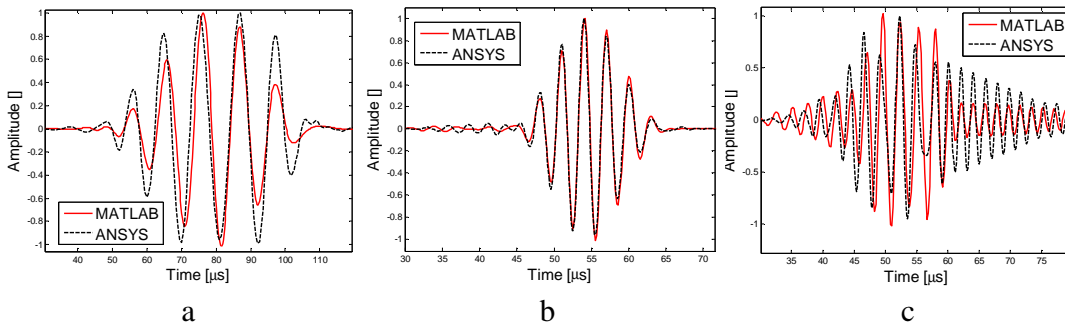


Figure 6. Amplitude - normalized signals generated by the IDT at: a) 100kHz, b) 330kHz c) 425kHz.

In figure 6 the amplitude-normalized time series obtained from point  $M_p$  (see figure 3) during the numerical tests are compared. At 330kHz both the waveform and normalized amplitude of the signal are almost identical (figure 6b). For the signals at 100kHz the shape of the waveform is similar but some differences in the amplitude may be observed (figure 6a). The worst results consistence can be observed at

frequency 425kHz where significant differences between the waveforms can be noticed (figure 6c).

## DISCUSSION

For all selected frequencies the results obtained from the simulations based on a frequency dependent transfer-function model are similar to the ones found with FEM analyses. The computational time required to determine the results when the proposed method was applied was two orders shorter than the period required to find the referential FEM results. For lower frequencies both beam patterns and waveforms show good agreement. At 425 kHz the differences are much higher, and they can be noticed especially for the time-history plots. The main source of these differences is the presence of a higher order mode S1 in the response of the structure [8] which is not modeled in the approximate method. Nevertheless, the shape of the beam pattern obtained from the developed technique is very similar to the one from FEM and can be used for the identification of the transducer properties.

Despite of the simplicity of the proposed method, developed model correctly simulates beam-pattern and time series not only in the frequency where dispersion is minimal but also in frequencies where its level was significant.

## CONCLUSIONS

The preliminary results which were obtained up to now show that the proposed method can be used for simulations of dispersive Lamb waves generated using complex shape transducers, i.e. IDTs. The method, due to its computational efficiency, can be used for the initial simulations to identify interesting frequencies or configurations of designed transducers. Subsequently, a detailed simulation can be performed for specific cases using FEM.

The main drawback of the proposed method, at the moment, is a lack of direct relationship between the transducer's material properties (i.e. piezoelectric constants, stiffness of used material, etc.) and the produced excitation. Moreover, only two fundamental modes are modeled in the system at this moment.

As future extensions of the model features, such as, mode excitability, higher order modes and material properties of the transducer are planned to be added to the model to increase its usefulness and accuracy.

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