ICSV19
Vilnius, Lithuania
July 08-12, 2012

AN INVESTIGATION OF A THEORETICAL TOOL FOR PREDICTING PERFORMANCE OF AN ACTIVE NOISE CONTROL SYSTEM

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Active noise control is a topic of increasing interest for car manufacturers. In the lower frequency range, active control could relax the need for sound insulating materials, leading to lighter vehicles with lower fuel consumption. Here, we investigate further the possibility of using multiple loudspeakers with the aim of enlarging the spatial zones where sufficient noise suppression is achieved. A model-based Linear Quadratic design of a linear multiple-output feedforward regulator is used to approximate known noise paths to multiple measurement positions by the combined secondary paths from the control loudspeakers. The attainable performance depends on the number and positioning of the control loudspeakers. Tools are needed for predicting and understanding the structural constraints on the attainable performance of the active control system. A measure of reproducibility of the primary paths between a noise source and a control area by a set of control loudspeakers has been investigated here. It uses the concept of an effective rank of the control system. With a given constellation of loudspeakers, the contribution of each loudspeaker to the possible attenuation of a disturbance has been analysed and the relationship between the reproducibility of the primary path and the achieved attenuation explored. The insights gained may result in a tool to help car manufacturers placing loudspeakers with both sound reproduction and active noise control in mind.

1. Introduction

Active noise control can be used to suppress undesired noise. It is of particular interest in the lower frequency range, where passive means of damping become less efficient. In the special case of a car interior, active noise control could be helpful in reducing undesired engine orders and road noise from the tires. As modern cars typically contain several loudspeakers, their sound systems could be used also for noise control; multiple loudspeakers give the possibility of extending the zone of control. We have shown that feedforward control designed by Linear Quadratic Gaussian (LQG) optimal control is an efficient model-based strategy for attenuating noise in an extended area using multiple loudspeakers [1], [2]. While the obtained results are promising, improved tools are needed for understanding the limits of performance inherent in a specific acoustic system and for understanding the effects of design parameters on the results. For example, a better way of designing the control signal penalties used in an LQG design would be desirable. At present, we utilize a simple criterion with a

This research has been supported by the Swedish Research Counsil under contract 90552701 and by Dirac Research AB. The authors also greatfully acknowledge the support from the Knut and Alice Wallenberg foundation (Dnr. KAW 2006.0152), which provided funding for acoustic measurement equipment.

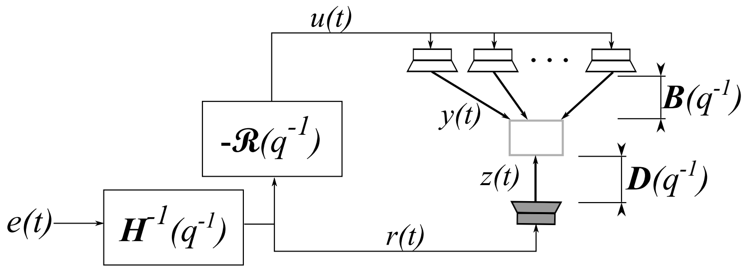


Figure 1: A linear acoustic system is affected by L disturbances $r(t)$ through the primary paths $\boldsymbol{D}(q^{-1})$ to the volume of interest. N loudspeakers, with secondary paths $\boldsymbol{B}(q^{-1})$, are used by the feedforward regulator $\boldsymbol{\mathcal{R}}(q^{-1})$ to control the sound field within the volume of interest.

diagonal input penalty matrix, with low gain in the operating frequency region of each loudspeaker and a high gain outside of it. Furthermore, the 'operating region' of a loudspeaker has been determined based on visual inspection of its frequency response. Better guidelines for the design of these penalties would hopefully lead to a regulator design that utilises the control energy more efficiently.

A theoretical tool that can answer questions such as those mentioned above, as well as predict the attainable performance of the regulated system has been investigated. The method studied uses a measure of reproducibility recently introduced by Lars-Johan Brännmark in [3], that answers the question 'how well can a specific target sound field be reproduced using the secondary loudspeakers?'. We will show here that this method can provide many theoretical insights into the performance of the feedforward LQG regulator. It can also help in the investigation of different loudspeaker setups, to find the one most advantageous to the problem at hand, without having to go through the regulator design for each setup. The regulator design, as discussed in Section 2.3, is computationally complex and requires non-trivial choices of design variables that influence the obtained performance.

1.1 Mathematical notation

Matrices of causal FIR filters are here represented in the time domain by matrices $\boldsymbol{P}(q^{-1})$ of polynomials in the backward shift operator q^{-1}, where $q^{-1}v(t) = v(t-1)$. The time-domain operator q^{-1} corresponds to z^{-1} or $e^{-j\omega}$ in the frequency domain. The transpose of a matrix $\mathbf{M}$ is denoted $\mathbf{M}'$. For a polynomial matrix $\boldsymbol{P}(q^{-1})$, the corresponding conjugate matrix $\boldsymbol{P}_*(q)$ is defined as its conjugate transpose, with the forward shift operator q substituted for q^{-1}. A rational matrix $\boldsymbol{\mathcal{R}}(q^{-1})$ of IIR filters will be represented by calligraphic symbols. Arguments are omitted below where there is no risk of misunderstanding.

2. Sound field reproducibility and LQG feedforward noise control

2.1 The acoustic system

A linear acoustic system is affected by L measurable disturbances, represented by a column vector $r(t)$, that are to be used for feedforward control, as illustrated in Fig. 1. The influence of these noise components is to be suppressed at M control points (measurement positions), using a set of N loudspeakers where we here assume $N \leq M$. The control points are assumed located in the volume to be silenced, spaced by distances less than the spatial Nyquist frequency of the highest frequency to be controlled. The channels from the N control loudspeakers to the M positions (variously denoted forward paths, secondary paths or control paths) are represented by the FIR matrix model

$$y(t) = \boldsymbol{B}(q^{-1})\, u(t) \quad . \tag{1}$$

Here, the $N \times 1$ vector $u(t)$ represents loudspeaker input signals at discrete time t, while the $M \times 1$ vector $y(t)$ is the sound sampled at the M control points.

The noise components to be controlled are described by noise paths (primary paths) from $r(t)$ modeled by an $M \times L$ FIR matrix,

$$z(t) = \boldsymbol{D}(q^{-1})\, r(t) \quad . \tag{2}$$

Spectral properties of $r(t)$ can furthermore be represented by a stable vector-autoregressive model

$$r(t) = \boldsymbol{H}^{-1}(q^{-1})\, e(t) \quad , \tag{3}$$

where the $L \times 1$ vector $e(t)$ is white, with zero mean and covariance $\mathbf{R_e}$.

2.2 Reproducibility

Denote by $\rho(m_1, m_2, \omega)$ the cross-correlation between the complex sound fields being generated by the system (1) at two particular control points, m_1 and m_2, at frequency $\omega/2\pi$. If the input $u(t)$ is white with covariance matrix $\mathbf{I}$, then the cross-spectral density matrix $\boldsymbol{B}(e^{-j\omega})\boldsymbol{B}_*(e^{j\omega})$ will, as shown in Section 2.8 of [3], describe all such correlations, with $\rho(m_1, m_2, \omega)$ being its element (m_1, m_2).

The symmetric positive semidefinite cross-spectral density matrix has M orthonormal eigenvectors $\Psi_m(\omega)$ and corresponding real-valued nonnegative eigenvalues $\sigma_m^2(\omega)$

$$\boldsymbol{B}(e^{-j\omega})\boldsymbol{B}_*(e^{j\omega})\Psi_m(\omega) = \sigma_m^2(\omega)\Psi_m(\omega), \quad m = 1, \cdots, M. \tag{4}$$

These constitute the left singular vectors and singular values, respectively, of $\boldsymbol{B}(e^{-j\omega})$. The singular vectors describe the principal components of the correlation function $\rho(m_1, m_2, \omega)$ and $\{\sigma_m^2\}$ represents their relative contribution, with $\sigma_j^2 > \sigma_{j+1}^2$. The singular vectors are spatially sampled approximations to the Karhunen-Loève expansion functions (see e.g. Chapter 10.6 in [4]) of a continuous-space correlation function. Since $\boldsymbol{B}(e^{-j\omega})$ has rank $\leq \min(M, N)$ and we here assume $N \leq M$, at most N of the singular values and the corresponding singular vectors are nonzero. These N gains and vectors are also called the *principal gains* and the *output principal directions* of the dynamic system [6]. Furthermore, some of the N nonzero singular values may be insignificant. The frequency-dependent *effective rank* N_r of the matrix $\boldsymbol{B}(e^{-j\omega})$ has been introduced in [3] for this reason, and can be defined as

$$N_r(\omega) = \arg\min_{N_r} \left\{ \frac{\sum_{m=N_r+1}^{N} \sigma_m^2(\omega)}{\sum_{m=1}^{N} \sigma_m^2(\omega)} < \delta \right\}, \tag{5}$$

where $\delta = 0.01$ will be used below. This represents the number of significant principal gains of the control system at frequency $\omega/2\pi$. The subspace spanned by $\Psi_{N_r+1}(\omega), \cdots, \Psi_N(\omega)$ represents the M-dimensional output directions $y(t)$ that are "hard to reach". Inputs having frequency $\omega/2\pi$ would produce little acoustic output at the control points in these directions, for all input signals of reasonable power. Typically, the effective rank increases with frequency and the area covered by $\{y\}$.

We now investigate how well a particular sound field, namely the one described by the noise model (2), with $r(t)$ white, can be controlled by a feedforward controller that uses only the components of the primary path represented by its effective rank. The target sound field $z(e^{-j\omega})$, for $\boldsymbol{H}(e^{-j\omega}) = \mathbf{I}$ and $\mathbf{R_e} = \mathbf{I}$, is orthogonally projected onto the space spanned by $\Psi_1(\omega), \cdots, \Psi_{N_r}(\omega)$,

$$\hat{\boldsymbol{D}}(e^{-j\omega}) = \sum_{m=1}^{N_r} \Psi_m(\omega)\Psi_m^H(\omega)\boldsymbol{D}(e^{-j\omega}). \tag{6}$$

The sound field $z(e^{-j\omega}) = \boldsymbol{D}(e^{-j\omega})e(e^{-j\omega})$ is then said to be reproducible by $\boldsymbol{B}(e^{-j\omega})$ at frequency ω with relative error $\alpha(\omega)$, $0 \leq \alpha(\omega) \leq 1$, if [3]

$$\frac{\|\boldsymbol{D}(e^{-j\omega}) - \hat{\boldsymbol{D}}(e^{-j\omega})\|_2^2}{\|\boldsymbol{D}(e^{-j\omega})\|_2^2} \leq \alpha(\omega). \tag{7}$$

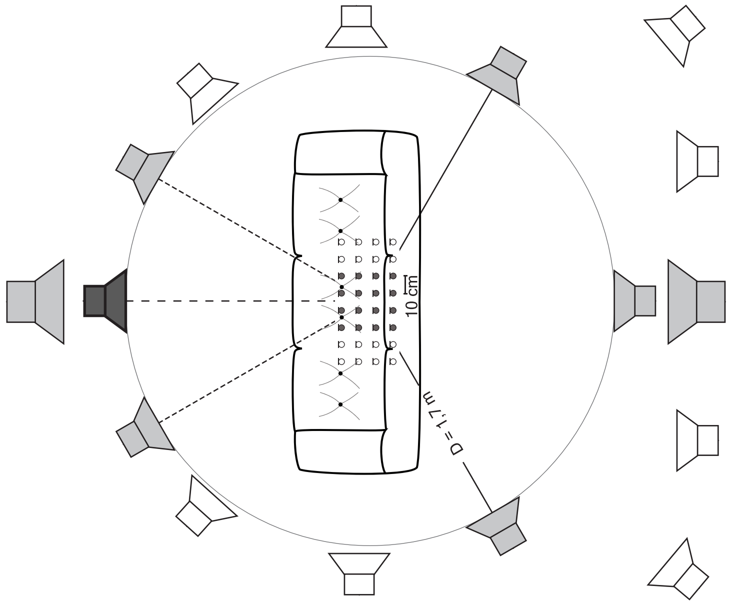


Figure 2: Experimental setup, 16 loudspeakers are located around a living room sofa, over which there are 16 inner microphone positions and 16 outer positions, both marked with '⌐○'. The loudspeaker used as noise source is marked as dark gray, the speakers marked as light gray mark one particular setup used. The two loudspeakers that are bigger represent the two subwoofers used, the other speakers are midrange.

2.3 LQG feedforward control

The LQG feedforward regulator is represented by a linear rational matrix

$$u(t) = -\boldsymbol{\mathcal{R}}(q^{-1})\, r(t+d) \quad , \tag{8}$$

of dimension $N \times L$ that operates on the measurable disturbances $r(t)$, resulting in a control error

$$\varepsilon(t) = z(t) + y(t) = (q^{-d}\boldsymbol{D} - \boldsymbol{B}\boldsymbol{\mathcal{R}})r(t+d) \quad . \tag{9}$$

The control paths $\boldsymbol{B}\boldsymbol{\mathcal{R}}$ are made to approximate the "target" $q^{-d}\boldsymbol{D}$ that represents the noise paths, or primary paths, delayed by d samples. Use of a longer so called *smoothing lag*, d, results in a higher approximation fidelity. However, to compensate for this the regulator (8) needs to act on a time-shifted (predicted) feedforward signal $r(t+d)$.

The model-based design of $\boldsymbol{\mathcal{R}}(q^{-1})$ aims to minimize the scalar quadratic criterion

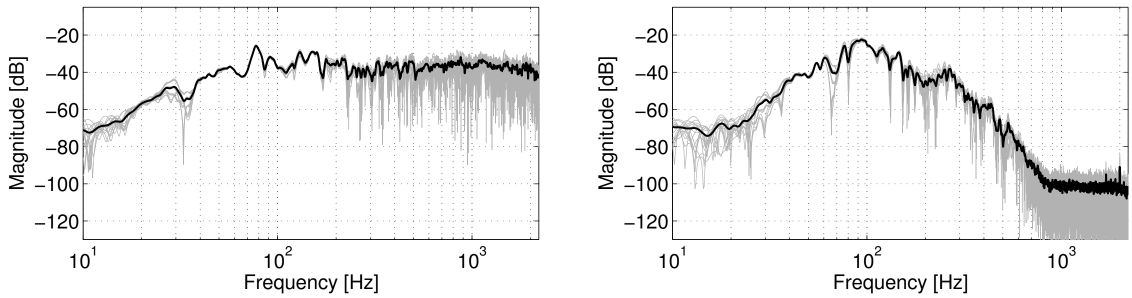
$$J \;=\; E\{(\boldsymbol{V}\varepsilon(t))'\boldsymbol{V}\varepsilon(t) + (\boldsymbol{W}u(t))'\boldsymbol{W}u(t)\} \quad , \tag{10}$$

under constraints of stability and causality of the matrix of feedforward controllers $\boldsymbol{\mathcal{R}}(q^{-1})$. The weighting $\boldsymbol{V}(q^{-1})$ is an $M \times M$ polynomial matrix of FIR filters of full normal rank M. The square polynomial matrix $\boldsymbol{W}(q^{-1})$ can be used to focus the control energy to appropriate frequencies, and into spatial subspaces that are appropriate for the room and the sound system. For example, when $\boldsymbol{W}(q^{-1}) = \text{diag}[W_j(q^{-1})]$, each scalar penalty FIR filter $W_j(q^{-1})$ can be given low gain within the operating range of loudspeaker j and high gain outside of that range.

The problem is a special case of a MIMO LQG feedforward regulator design problem in input-output form discussed in [5]. The solution is obtained by first computing an $N \times N$ polynomial matrix $\boldsymbol{\beta}(q^{-1})$, that has a stable and causal inverse, from the polynomial matrix spectral factorization equation

$$\boldsymbol{\beta}_*\boldsymbol{\beta} = \boldsymbol{B}_*\boldsymbol{V}_*\boldsymbol{V}\boldsymbol{B} + \boldsymbol{W}_*\boldsymbol{W} \quad . \tag{11}$$

Such a matrix is guaranteed to exist under mild conditions, for example by the use of a penalty matrix $\boldsymbol{W}$ such that $\det[\boldsymbol{W}(z^{-1})] \neq 0$ on the unit circle.



(a) Noise source

(b) Subwoofer

Figure 3: Frequency responses from the noise source to the microphone positions and from the subwoofer placed behind the sofa. The grey curves show the individual frequency responses and the black curve shows their RMS average.

A causal $N \times L$ polynomial matrix $\boldsymbol{Q}(q^{-1})$ is then, together with an $N \times L$ polynomial matrix $\boldsymbol{L}_*(q)$, obtained as the unique solution to the polynomial matrix Diophantine equation

$$q^{-d}\boldsymbol{B}_*\boldsymbol{V}_*\boldsymbol{V}\boldsymbol{D} = \boldsymbol{\beta}_*\boldsymbol{Q} + q\,\boldsymbol{L}_*\boldsymbol{H} \quad . \tag{12}$$

The unique stable linear regulator (8) that minimizes the criterion (10) for the model (1), (2), (3) is then the IIR filter

$$u(t) = -\,\boldsymbol{\beta}^{-1}(q^{-1})\,\boldsymbol{Q}(q^{-1})\,r(t+d) \quad . \tag{13}$$

See Section 3.3 of [5] for a proof and for conditions on the polynomial degrees of $\boldsymbol{Q}(q^{-1})$ and $\boldsymbol{L}_*(q)$. In the experiments described below, we truncate the impulse responses to where they contain significant energy and realize the controller with high order FIR filter approximations.

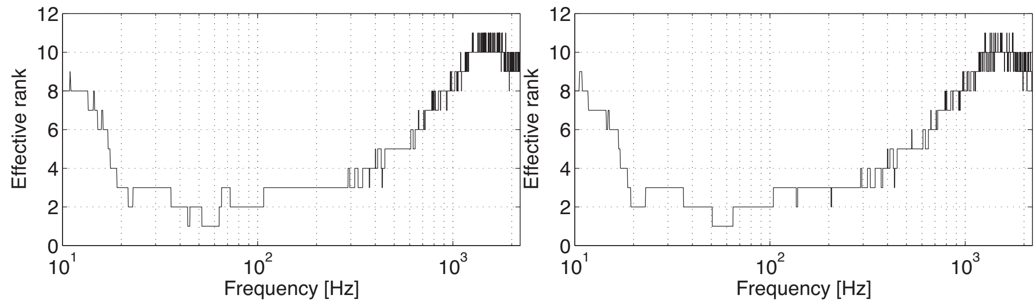
2.4 Comments on the reproducibility measure and LQG feedforward control

The measure of reproducibility (7) is based on cross-correlations, or second order statistics, of the sampled spatial sound field. This property makes the reproducibility interesting to compare to the results obtained from an LQG design, since the LQG design criterion is quadratic and the utilized feedforward regulator is linear, making the method sensitive only to the second-order statistics. This similarity in the basic assumptions between the two methods gives us the hope that the measure (7) is suitable as an indication as to how well sound field control by LQG will perform. Another indication that sound field control by LQG might agree with the reproducibility measure (7) is that an LQG design that uses a nonsingular but small control penalty matrix $\boldsymbol{W}$ tends to regulate the system in an approximate MSE sense, but without using unrealistic input levels. Using an orthogonal projection of the sound field onto the subspace reached by 99% of the control energy is a similar way of indicating how much of that sound field can be reproduced using reasonable control energy levels.

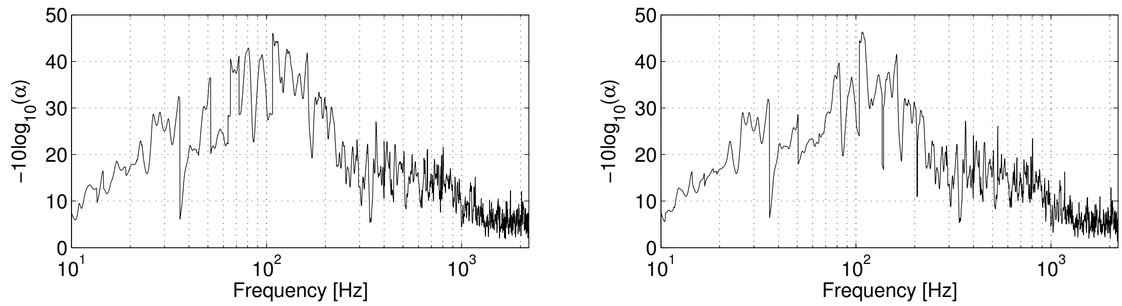
Since $\alpha(\omega)$ in (7) is the relative difference between the target and the reproduction of the target by the secondary loudspeakers, it can be interpreted as the frequency dependent (mean) attainable attenuation over the area to be silenced. Therefore, in the evaluation, we will compare $-10\log_{10}(\alpha(\omega))$ to the actually obtained mean attenuation on a dB scale .

3. Experimental evaluation

All experiments were performed in a room of dimensions $4.6 \times 6 \times 2.6$ m, which offers better possibilities to vary the positions of loudspeakers than a car interor. Around a living room sofa in the middle of the room, 14 midrange loudspeakers, and two subwoofers were placed, as indicated in Fig. 2. The impulse responses from each loudspeaker were measured at 32 microphone positions over



(a) Effective rank using 15 control loudspeakers. (b) Effective rank using 12 control loudspeakers.

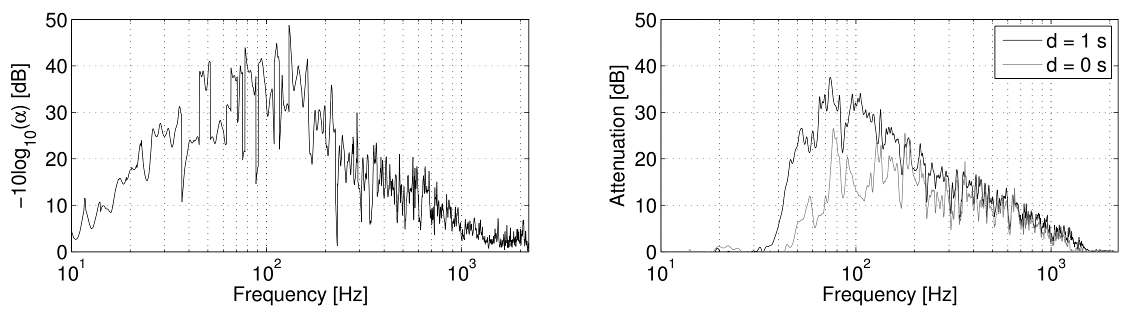


(c) Reproducibility of the target using 15 loudspeakers. (d) Reproducibility of the target using 12 loudspeakers.

Figure 4: Comparison of the effective rank and reproducibility when using 15 loudspeakers and when using 12. There is little difference between the two setups.

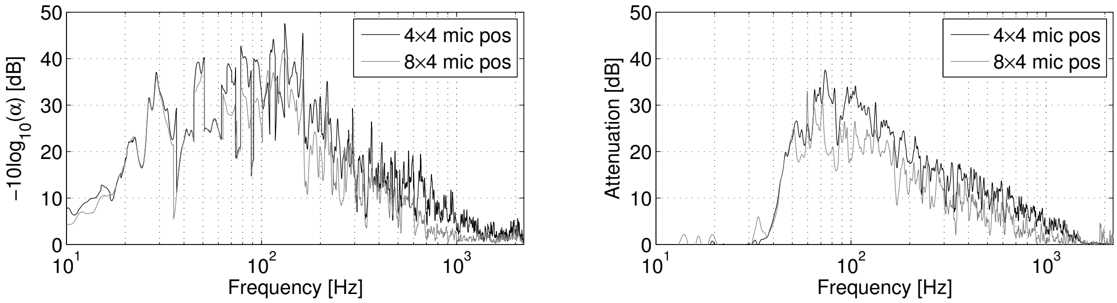
the sofa at ear height. Of these, only the 16 marked gray were regarded in all but the last experiment, where all the microphone positions where used. One of the speakers, placed directly in front of the sofa, was designated noise source, and will be referred to as the noise speaker. The transfer function matrix $\boldsymbol{D}(e^{-j\omega})$ from the noise speaker to the microphone positions will be referred to as the target, or target transfer function. The other speakers were available as, in total 15, control loudspeakers. The frequency responses of the target transfer functions for the 16 gray measurement positions in Fig. 2 are shown in Fig. 3a, together with the RMS average. In Fig. 3b, the frequency responses and the RMS average of the subwoofer placed behind the sofa, directly opposite of the noise speaker, is shown. Several combinations of control loudspeakers were evaluated. The effective rank of the system for the different configurations was calculated, and the reproducibility of the target transfer function matrix was investigated.

One immediate advantage of evaluating the effective rank of $\boldsymbol{B}$ can be seen in Fig. 4. In Fig. 4a, the effective rank has been calculated for all 15 control loudspeakers. Using 15 loudspeakers, the



(a) Reproducibility of the target. (b) Obtained attenuation for smoothing lag $d = 1$ s (black curve) and $d = 0$ s (gray curve).

Figure 5: Comparison of the reproducibility of the target with the obtained RMS average over 16 microphone positions of the attenuation with different smoothing lag using 7 control speakers.



(a) The reproducibility of the target for the two areas. (b) The obtained attenuation for the two areas.

Figure 6: Comparison of the reproducibility of the target with the obtained attenuation for different sizes of the controlled area. The black curves show a 0.3×0.3 m area, the gray curves show a 0.7×0.7 m area.

theoretical limit for the effective rank is 15. In this case however, Fig. 3b shows that the two subwoofers are not able to contribute to the sound field at the highest frequencies, leaving 13 as the limit for the effective rank there. Inspection of Fig. 4a shows the maximum to be 11 up to 2000 Hz[1]. The interpretation of this is that there are two loudspeakers in the setup that do not contribute to the sound field in a way that the other 11 couldn't do on their own. Theoretically, these two loudspeakers could be removed without changing the set of sound fields that could be reproduced by the system. Furthermore, the effective rank at lower frequencies is fairly low, leading to the suspicion that also the two subwoofers may be removed. This is illlustrated in Fig. 4b where the two subwoofers, along with one midrange loudspeaker have been removed with little effect on the effective rank. This lack of effect when removing loudspeakers from the system can also be seen by comparing Figs. 4c and 4d that show the reproducibility $-10\log_{10}(\alpha)$ of the target by the two systems. This saturation of the number of loudspeakers needed to approximate a sound field in a particular spatial region and frequency range would be more difficult to discover by experimenting with various controller designs.

The high variability of the reproducibility in the low frequency region visible in Figs. 4c and 4d can be explained by jumps in the effective rank, which is an integer quantity. These jumps have large impact at the lower frequencies and smaller impact at higher frequencies since a jump from effective rank one to effective rank two means using two basis functions instead of one in the projection of the target sound field. This has a much larger impact than going from, say, eight to nine basis functions.

Cancellations with several different setups were performed to evaluate how well the measure of the reproducibility of the target sound field corresponds to the obtained attenuation of an actual cancellation. For this purpose, seven of the control loudspeakers were used, marked as light gray in Fig. 2, with different sets of design variables used for the LQG feedforward controller design. Shown in Fig. 5a is the reproducibility of the target by the system of loudspeakers. For all the designs the penalty matrix $\boldsymbol{V}$ from Eq. (10) was set to the unit matrix and the control loudspeaker power penalty matrix $\boldsymbol{W}$ was set to be diagonal, with a low gain in the frequency region of interest for a particular loudspeaker and a high gain outside of this region. The frequency region of interest was chosen as $[35\ 400]$ Hz for the subwoofers, while it was chosen as $[40\ 1500]$ Hz for the midrange speakers.

In Fig. 5b, the obtained attenuation is shown for two different (extreme) values of the smoothing lag d, $d = 1$ s and $d = 0$ s. The higher of the two values was chosen sufficiently high so that a further increase in d would not yield an improved result. From the figure, it can be seen that with a high smoothing lag the attenuation curve follows the trend predicted by the reproducibility curve in Fig. 5a well from 50 to 1500 Hz. When the smoothing lag is reduced to $d = 0$ s, the attenuation is largely

[1]Note in Figs. 4a and 4b that the effective rank increases at the lowest frequencies. This is due to noise in the measurements, that is most prominent in this fequency region. The noise gives the impression that the sound field is more complex than it actually is. At low frequencies, the complexity of the sound field actually decreases because of longer wavelengths. Therefore the (true) effective rank will be low in the lowest frequency region.

unaffected for frequencies above 300 Hz, but is reduced in the lower frequency region.

With respect to the volume or area under control, the effective rank of the system can help explain how the attenuation changes when the area is changed. In Fig. 6, the reproducibility and the attenuation are shown for two different areas under control. When the area is enlarged, both the reproducibility and the attenuation are reduced to the same extent. The reduction in reproducibility can be explained by the increased complexity of the sound field. The effective rank will increase for the larger system as compared to the smaller. However, since there are twice as many microphone positions, the number of basis functions needed has increased even more. Therefore, the increase in effective rank that has occured is not enough, and the reproducibility of the target will decrease.

4. Conclusions

The effective rank of the control paths and the reproducibility of a target sound field by the control system provides insights that would otherwise be difficult to obtain. Loudspeakers that will not contribute anything extra to a sound field can be isolated and removed. The measure of reproducibility will give a fair description of how well the control loudspeakers can reconstruct the target sound field. However, there are some gray zones where the method studied here is unable to fully describe what is going on. The measure of reproducibility will give an indication of the attainable performance only when the smoothing lag d in the design of the feedforward regulator is chosen relatively high. The reproducibility measure is not adequate to describe what happens (or why) at low smoothing lags. To explain this, a theory describing how time-domain properties of compensators affect their frequency-domain ability to approximate a target function is needed.

When it comes to the input penalty matrix $\boldsymbol{W}$, the effective rank gives a hint about how to design it to better use the input energy. Knowing what directions in the space spanned by the singular vectors of $\boldsymbol{B}(e^{-j\omega})$ that carry little or no energy, $\boldsymbol{W}$ can be constructed so as to contain the effective nullspace of $\boldsymbol{B}$, thus penalising those directions.

Finally, this study shows that control using feedforward LQG is a method well suited for sound field synthesis. The attenuations actually achieved by the method accord well with those predicted by analysis of the attainable reproducibility of the target sound field.

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