

## ABSTRACT

Monitoring of large plate structures using Lamb waves requires normally a dense network of piezoelectric sensors. Active ultrasonic arrays, due to their superior sensitivity and beam steering capability are an interesting alternative to sensor networks. In this paper we present a minimum variance distortionless response (MVDR) approach to beamforming of Lamb waves using a single transmitter and a uniform rectangular array (URA) for reception. Dispersion effects are compensated using theoretically calculated and experimentally verified dispersion curves. The combination of the MVDR approach and the 2D array allows for suppression of spurious Lamb modes. The proposed algorithm is evaluated using selected experimental data.

## 1. INTRODUCTION

Structural health monitoring (SHM) techniques for plate structures using Lamb waves require normally a dense network of transducers covering the whole structure. An alternative approach, which is the subject of this paper is based on the use of active smart arrays capable of monitoring a larger areas of the plate.

In this paper we present experimental results obtained using the adaptive minimum variance distortionless response (MVDR) approach for Lamb wave imaging. The MVDR beamformer is one of the most commonly used adaptive beamformers. The basic steps of the proposed method follows the approach used in medical ultrasound by Sasso [1], Synnevåg [2] and others. Here, the technique is extended to handle dispersion and two-dimensional arrays. The motivation for using this approach is its ability to adaptively suppress interfering signals. Besides allowing 360° azimuthal coverage, the two dimensional array in conjunction with the MVDR technique enables adaptive suppression of spurious Lamb modes.

We consider monitoring of a thin plate through a pulse-echo ultrasonic imaging using a rectangular array that covers 360° in the setup shown in Fig. 1. A

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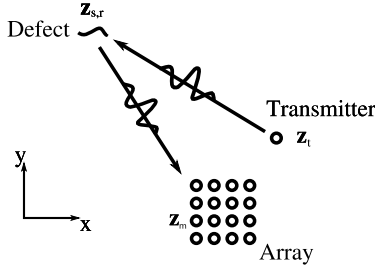


Figure 1. Overview of the imaging problem.

single transmitter, either an array element or a separate transducer, located at  $\mathbf{z}_t = [x_t, y_t]^T$  and excited with a short pulse or a windowed sinusoid, insonifies the plate. The back-scattered signal,  $g_m(t)$ , is acquired from each array element  $m$ , either simultaneously or multiplexed through repeated transmitter excitation. The objective is to estimate the power originating from each point  $\mathbf{z} = [x, y]^T$  in the region of interest.

## 2. THEORETICAL BACKGROUND

The main advantage of the delay-and-sum (DAS) beamformer, which is the most commonly used technique for beamforming is its robustness, which comes at the cost of poor performance compared to more advanced methods. The DAS beamformer uses predetermined weights on the input signals and is therefore independent of the received data. To place the proposed approach into context, this section gives a brief review of the DAS and MVDR methods for narrowband far-field beamforming for 2D arrays.

The output from a narrowband beamformer is  $y(t) = \mathbf{w}^H \mathbf{g}(t)$ , where  $H$  is the conjugate transpose,  $\mathbf{w}$  is the complex valued weight vector, and  $\mathbf{g}(t)$  is a vector containing the narrowband input signals from each array element at time  $t$ .

For the narrowband case the DAS weight vector is simply set to the steering vector  $\mathbf{a}(k_x, k_y)/M$ , where  $M$  is the number of array elements. It is also common to apply some window function (apodization) to give different weights to the elements. The unwindowed output of the standard beamformer is  $y_{BF}(t, k_x, k_y) = \frac{1}{M} \mathbf{a}^H(k_x, k_y) \mathbf{g}(t)$ . Without any constraints, the DAS is simply a filter with unit gain for the signals with wavenumber  $(k_x, k_y)$ .

The MVDR method, also known as Capon's method, sets a data dependent weight vector, which minimizes the influence of interfering signals while passing the desired signal undistorted. The method yields a filter that minimizes the output power from the array,  $\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w}$ , under constraint that the gain for the signals with wavenumber  $(k_x, k_y)$  is 1,  $\mathbf{w}^H \mathbf{a}(k_x, k_y) = 1$ , where  $\mathbf{R} = E\{\mathbf{g}(t)\mathbf{g}^H(t)\}$  is the covariance matrix, which in practice has to be replaced by the sample covariance matrix  $\hat{\mathbf{R}} = \frac{1}{N} \sum_{t=1}^N \mathbf{g}(t)\mathbf{g}^*(t)$ .

This means that the filter will place nulls that minimize the influence of the

interferers. The solution to this optimization problem is simply [3]

$$\mathbf{w} = \frac{\hat{\mathbf{R}}^{-1}\mathbf{a}(k_x, k_y)}{\mathbf{a}^H(k_x, k_y)\hat{\mathbf{R}}^{-1}\mathbf{a}(k_x, k_y)}. \quad (1)$$

The estimated power of the signal with wavenumber  $(k_x, k_y)$  is

$$P_{MVDR}(k_x, k_y) = \frac{1}{\mathbf{a}^H(k_x, k_y)\hat{\mathbf{R}}^{-1}\mathbf{a}(k_x, k_y)}. \quad (2)$$

A major disadvantage of the MVDR algorithm is its sensitivity to errors in the steering vector, where even small errors can lead to cancellation of the desired signal. Another issue that needs to be addressed is that the MVDR is incapable of handling highly correlated sources, which also may result in the signal cancellation.

### 3. METHODS

To overcome the limitations concerning narrowband assumptions, a number of preprocessing procedures have been proposed in the literature, for example, the coherent subspace approach [4] and the steered covariance matrix [5]. The signal cancellation effect of the MVDR, which is a consequence of highly correlated sources, has also been addressed in previous research with methods such as the coherent subspace approach and spatial smoothing [6].

Here, we investigate a combination of the steered covariance matrix and the spatial smoothing approaches, more details can be found in [7].

#### 3.1. Focusing

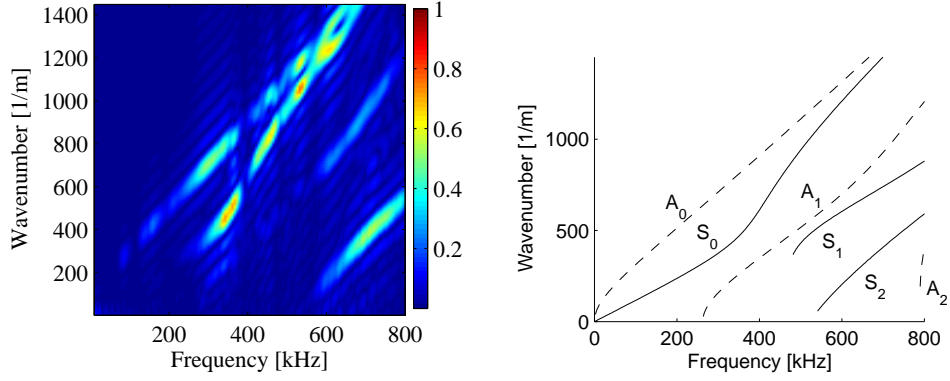
Focusing, or steering, the data to a certain spatial position is used as a remedy for the problem with a frequency dependent steering vector, which also allows a straightforward treatment of the dispersion. The incoming signal is dispersion and divergence compensated and transformed into the spatial domain.

#### 3.2. Spatial Smoothing

The most significant limitation of the MVDR algorithm for array imaging applications is its inability of treating coherent signals, which can lead to signal cancellation for multiple scatterers. In this work spatial smoothing [6] is used to address this issue.

#### 3.3. Robustness

The MVDR algorithm, when used on the signal covariance matrix, is very sensitive to errors in the steering vector  $\mathbf{a}(k_x, k_y)$ . Steering vector errors can be caused by uncertainties in element position, physical differences between array elements, or errors in the estimated dispersion characteristics of the plate. The simplest and most common regularization method is diagonal loading, which is performed by adding a diagonal matrix  $\alpha\mathbf{I}$  to the sample covariance matrix  $\hat{\mathbf{R}}$ .



**Figure 2.** Experimentally (left) and theoretically (right) determined dispersion curves for the 6 mm aluminum plate.

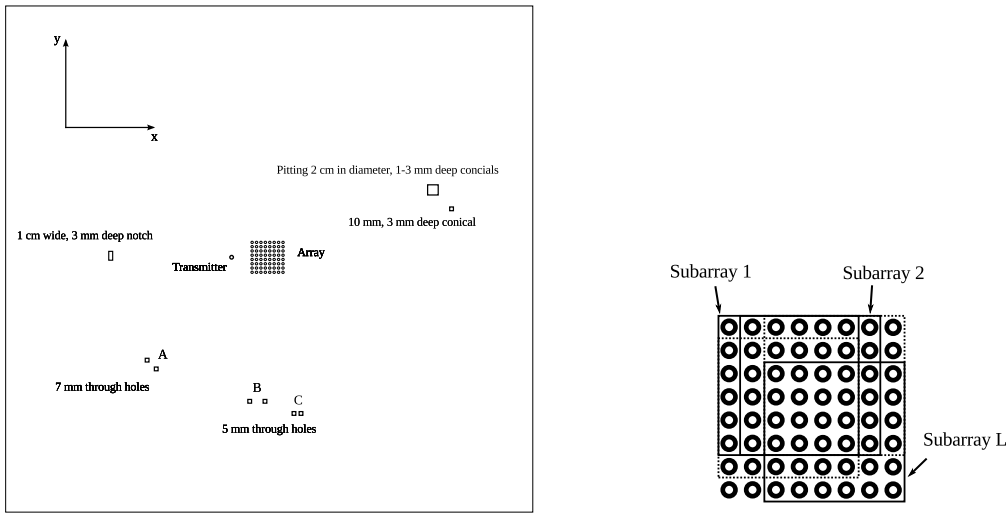
#### 4. EXPERIMENTAL RESULTS

The proposed method was evaluated on experimental data and compared to the DAS beamformer. The object used for the evaluation was a 6 mm thick, 750x750 mm aluminum plate (6082-T6) with artificial defects. The artificial defects included pairs of drilled through holes, a 1 cm wide notch, and some artificial pits with depths 1 to 3 mm. The layout of the defects and the positions of the array and the transmitter are shown in Fig. 3. The pairs of holes are labeled A, B and C. Pair A consists of two holes, 7 mm in diameter, 28 mm apart while pair B and C consists of two 5 mm holes each, located 21 and 10 mm apart, respectively.

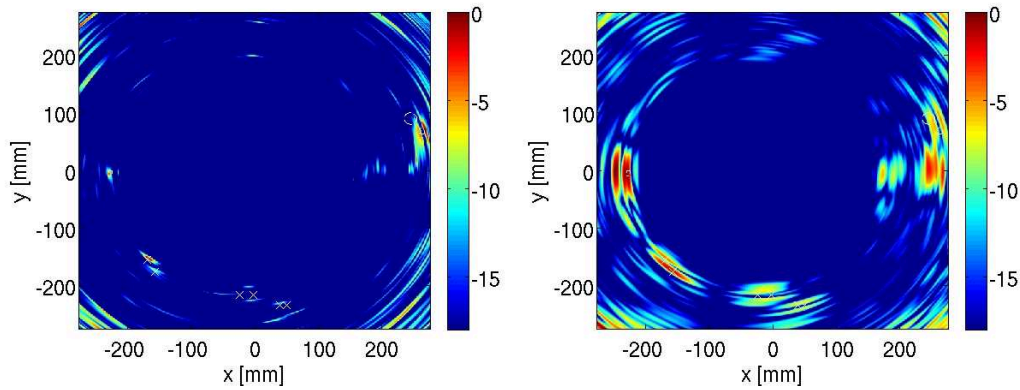
A single stand-alone pinducer was used for the excitation and the back-scattered signals were received by an 8x8 element rectangular array A1 with element spacing 3.5 mm, which was built of *pinducers* from Valpey Fisher Corp. (diam. 1.5 mm and resonance frequency of 1.1 MHz). The pinducers were coupled to the plates through a thin layer of oil which limited the detection and generation to the out-of-plane displacement. Spatial smoothing was applied on all results for the MVDR approach using 9 overlapping 6x6 subarrays and thereby making the effective array size 6x6 (see Fig. 3). The DAS approach did not benefit from spatial smoothing, the full array was used instead. The signal focusing was performed using frequencies between 50 and 450 kHz.

A single transmitter/receiver pair was used to experimentally determine the amplitude of the received Lamb modes and to validate the calculated dispersion curves. The spatio-temporal FFT resulted in a multi-mode signal presented in the frequency-wavenumber power spectrum in Fig. 2. The theoretical dispersion curves, also shown in Fig. 2, matching the measured dispersion curves, were calculated using the longitudinal wave velocity 6198 m/s and the transverse wave velocity 3158 m/s.

Ideally, all received modes are processed and used as a basis for the evaluation of the structure. Due to the shorter wavelength of the  $A_0$  mode compared to the  $S_0$  mode, and the poor agreement between simulated and measured dispersion characteristics for the  $S_0$  mode, only results from focusing using the  $A_0$  mode dispersion characteristics are presented here. For the frequency range used in



**Figure 3.** Layout of the inspected aluminum plate. The labels A,B,C identify three pairs of closely spaced holes (left). Spatial smoothing – the array is divided into  $L$  overlapping subarrays (right).



**Figure 4.** Measurement results from 6 mm plate for  $\epsilon = 50$ . MVDR (left) and DAS focusing on the  $A_0$  mode (right). True positions of the defects are indicated by white crosses (holes, lower part of the image), and dashed lines (pittings and notch, at  $0^\circ$  and  $180^\circ$ , respectively). Log scale cut at -15 dB.

the experiments the  $A_0$  mode had wavelengths down to 7 mm. To avoid spatial aliasing, this required an array element spacing of a maximum of 3.5 mm, which was satisfied by Array 1. Using larger array with higher element spacing while maintaining the same number of elements as Array 1, would improve the resolution but could lead to aliasing effects.

The input signal was generated by a HP8116 function generator and was a single square pulse, 1  $\mu$ s long with amplitude 16 V. The array pinducers were connected to an Agilent Infiniium oscilloscope through a custom built multiplexing box followed by an AD8335 amplifier from Analog Devices. The sampling rate of the oscilloscope was set to 25 MHz. Due to the limited resolution of the oscilloscope (8 bits), the received edge reflections had to be saturated to get sufficient resolution of the weaker defect reflections. The received signal from each element was averaged 16 times and the direct signal from the transmitter to the array was removed before processing the signals.

An example enabling the comparison of the DAS and the MVDR approaches using Array 1 can be seen in Fig 4. The log scale is cut at -15 dB due to poor

SNR. The holes are well pronounced in the lower part of the images where their true positions are marked with white crosses. The pit and the notch are seen at  $0^\circ$  and  $180^\circ$ , respectively.

The saturation of the edge reflections created artifacts in post-processed data from both algorithms in the areas closer than 100 mm to the edges, which is the reason for not showing images covering the whole plate.

From the experimental results presented here it is apparent that the MVDR approach performs much better than the DAS in terms of resolution, sidelobe suppression and spurious mode suppression. For some cases the MVDR has problems with signal cancellation which leads to underestimation of power.

## 5. CONCLUSIONS

A method for adaptive beamforming of Lamb waves has been presented in the paper. The dispersion of the Lamb modes was compensated by using theoretically predicted dispersion curves. Dispersion compensated data was processed using both standard DAS beamformer and the MVDR beamformer. Experimental results show that the MVDR can yield better performance compared to the standard DAS approach in terms of higher resolution and better suppression of spurious Lamb modes. Signal cancellation is an issue that needs to be addressed when working with the MVDR algorithm. Increasing the number of subarrays used for spatial smoothing can reduce signal cancellation effect and simultaneously increase robustness at the cost of lower resolution.

The MVDR algorithm, presented in the paper can be easily generalized to the situation where one or a group of array elements of the receiving array is also used as a source of Lamb waves.

## ACKNOWLEDGMENTS

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