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Integer Bits-Optimal Power Allocation Scheme with Proportional Fairness in OFDMA

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Abstract - **A subcarrier-power-integer bits allocation scheme is proposed that is aimed to maximize the throughput of downlink OFDMA systems under constraints on both total available power and the proportional fairness among users. The scheme is implemented in a two-step allocation procedure: initial integer bits allocation, and power reallocation. One distinctive feature of the proposed scheme is the reallocation of the remaining power due to the allocation of initial *integer* bits to subcarriers, which leads to significantly increasing the system's throughput. Simulations are made to evaluate the throughput as a function of SNR and active users' number, respectively, the rate proportionality as a function of the users' number, and the variation of the used power versus SNR. Comparison of the proposed scheme is carried out with two existing non-integer bits allocation schemes without and with considering rounding off. The comparison of simulation results shows that the proposed scheme can achieve the similar performance to the two schemes' (without rounding -off included), and the better performance than the two schemes with rounding off, in terms of system throughput.**

1. Introduction

Orthogonal frequency division multiple accesses (OFDMA), or a multiuser OFDM, is a very attractive modulation scheme and a multiple access method for 4G wireless networks [1],[2]. In an OFDMA system, the available bandwidth is divided into a set of subchannels (or subcarriers), which can be dynamically shared by all active users. Since the fading situations that each user undergoes may be *independent*, the probability that all users experience deep fading on the same subcarriers is very low. Thus the users may get the allocation of subcarriers without suffering deep fading, yielding the multiuser diversity. Exploiting the multiuser diversity, one can improve the total throughput by adapting the allocation of subcarriers, bits and power according to the users' channel states.

Optimal resource allocation for OFDMA systems in downlink or/and uplink transmission is based on throughput maximization or transmit power minimization [3]-[6]. For example, in [3], an algorithm for the minimization of the total transmitted power is proposed under a fixed BER requirement and a given set of user data rates. In the algorithm, Lagrange optimization method is used and the parameters are "relaxed" on non-integer. However, the algorithm requires a large number of iterations to converge and it does not converge smoothly. In [4]-[6], the system throughput is maximized under the constraint of rate proportionality between different users.

However, in [3]-[6], the bit allocation solution is generally non-integer; therefore, a suboptimal integer rounded bits allocation is needed [7]-[9]. The errors caused by the rounding-off should not be ignored in many cases. Thus the tradeoff between the power consumption and throughput change needs be considered during the rounding-off procedure. In [7], a rounding algorithm is introduced based on the 'quality factors' of the subcarriers and it can result in significant power saving for DMT systems. In [8] and [9], integer bit allocation is considered for DMT application, but the rates proportionality is not considered.

In this paper, we focus on the development of integer bits allocation method which maximizes the throughput of the downlink OFDMA system under the constraints of total transmit power, desired bit error rate (BER) and rate proportionality (proportional fairness) among different users.

The rest of this paper is organized as follows: in Section 2, the throughput maximization problem considered in this paper is formulated; in Section 3, the subcarriers-power-integer bits allocation scheme that maximizes the throughput of the system is developed. Numerical results for the proposed scheme and the comparison with the methods in [4] and [5] with and without rounding off are provided in Section 4. Section 5 concludes the paper.

2. Formulation of the Integer Bits Allocation Problem

Assume there are K active users in an OFDMA system which divides the available bandwidth B into N subcarriers. The gain of the nearly time-invariant channel seen by the kth user on the nth subcarrier, $|h_{k,n}|^2$, is assumed to be perfectly known at the transmitter. Then the number of bits, $b_{k,n}$, of the kth user assigned to the nth subcarrier can be determined based on the signal-to-noise ratio (SNR) $\gamma_{k,n}$ at the receiver and the target BER, that is,

$$b_{k,n} = \Phi(\gamma_{k,n}) \tag{1}$$

where $\Phi(\gamma_{k,n})$ is the rate-power function which depends on the target BER, the noise statistical feature, baseband modulation, etc. The SNR $\gamma_{k,n}$ is

$$\gamma_{k,n} = p_{k,n}\frac{\left|h_{k,n}\right|^2}{N_0\frac{B}{N}} = p_{k,n}H_{k,n} \tag{2}$$

where $p_{k,n}$ is the transmit power allocated to user k on subcarrier n. N_0 is the variance of the additive white Gaussian noise. $H_{k,n} = \left|h_{k,n}\right|^2 / (N_0\frac{B}{N})$ is the channel gain to noise power ratio(CNR).

When noise is assumed to be additive white Gaussian, and a square M-QAM with Gray bit mapping is used in the baseband modulation, the BER as a function of $\gamma_{k,n}$ and $b_{k,n}$ can be approximated to within 1dB for $b_{k,n} \geq 4$ and $\text{BER} \leq 10^{-3}$ as [5]

$$\text{BER}_{MQAM}(\gamma_{k,n}) \approx 0.2\exp\left[\frac{-1.6\gamma_{k,n}}{2^{b_{k,n}}-1}\right] \tag{3}$$

Solving for $b_{k,n}$, the logarithmic expression for the rate-power function can be obtained by "gap-analysis", in the following manner,

$$b_{k,n} = \Phi(\gamma_{k,n}) = \log_2\left(1+\frac{\gamma_{k,n}}{\Gamma}\right) \tag{4}$$

where SNR gap, $\Gamma = -\ln(5\text{BER})/1.6$, is a function of the target BER, which indicates the gap of SNR that is needed to reach a certain capacity between practical implementations and the information theoretical results. Note that $b_{k,n}$ in general is non-integer, and it is not attainable practically. In a practical system, the rounding-off of non-integer bits into integer ones is necessary [7]-[9]. This means that a new constraint, namely, integer bits allocation, should be considered.

Then the throughput maximization problem can be formulated as,

$$\max_{p_{k,n},\rho_{k,n}} R = \max_{p_{k,n},\rho_{k,n}} \sum_{k=1}^{K}\sum_{n=1}^{N}\rho_{k,n}b_{k,n} \tag{5}$$

subject to:

$$\sum_{k=1}^{K}\sum_{n=1}^{N}p_{k,n} \leq P_{total}, \text{and } p_{k,n} \geq 0, \forall k, \forall n \tag{5a}$$

$$\rho_{k,n} = \{0,1\}, \forall k, \forall n, \ \sum_{k=1}^{K}\rho_{k,n} = 1, \forall n \tag{5b}$$

$$R_1 : R_2 : \cdots : R_K = \gamma_1 : \gamma_2 : \cdots : \gamma_K \tag{5c}$$

$$b_{k,n} \in \{0,1,\cdots,b_{\max}\} \tag{5d}$$

where P_{total} is the total available power for transmission over all subcarriers. $\gamma_1,\gamma_2,\cdots,\gamma_K$ are the predefined proportions of total bit rates of all users, and $R_1,R_2,\cdots,R_K$ are the bit rates for all K users, respectively. $\rho_{k,n}$ has the value 0 or 1, where $\rho_{k,n}$=1 if subcarrier n is allocated to user k, and $\rho_{k,n}$=0, otherwise. $b_{\max}$ (an integer) corresponds to the maximum allowable size of QAM constellations. The constraint imposed on integer $b_{k,n}$ in (5d) makes the optimizing problem in this paper different from that in [4]-[6]. The issue on the constraint on $b_{k,n}$ (5d) is dealt with in the following section.

3. Subcarrier-power-integer bits allocation scheme

It can be seen from (5) that, to achieve maximum throughput, one can optimize parameters, $p_{k,n}$ (or $b_{k,n}$) and $\rho_{k,n}$ subject to the constraints (5a)-(5d). Since $b_{k,n}$ is a nonlinear function of $p_{k,n}$, then the problem itself is a nonlinear constrained optimizing problem with integer variables.

Since $b_{k,n}$ obtained from (4) can not be guaranteed to be an integer, and in a practical system only integer bits can be allocated to subcarriers, then an additional step is needed to perform rounding-off operation in order to obtain integer bits. In the proposed scheme, integer bits are directly allocated to subcarrries based on

$$b_{k,n} = \min\left\{b_{\max}, \left\lfloor \log_2\left(1+\frac{\gamma_{k,n}}{\Gamma}\right)\right\rfloor\right\} \tag{6}$$

which is modified from (4). Here the floor function $\lfloor x \rfloor$ means rounding x to the nearest integer smaller than x.

Assume that an initial subcarriers-bit-power allocation scheme is used to allocate the subcarriers to all active users and to assign integer bits to each subcarrier based on (6) according to its channel states (i.e., CNR). The integer bits allocated in this way is smaller than the non-integer bits obtained from (4). Therefore, the actually used power per subcarrier will be often less than the power initially allocated to the subcarriers, which means that there is some power left after the initial subcarriers-power-bit allocation.

From (2) and (4), it follows that the total used power after the initial resource allocation becomes

$$\begin{aligned} p^{(U)} &= \sum_{k=1}^{K}\sum_{n=1}^{N}p_{k,n}^{(U)} \\ &= \sum_{k=1}^{K}\sum_{n=1}^{N}\left(2^{\rho_{k,n}b_{k,n}}-1\right)\Gamma / H_{k,n} \end{aligned} \tag{7}$$

Then the remaining power will be

$$p^{(A)} = P_{total} - p^{(U)} \tag{8}$$

which can be reallocated to some users so as to increase the throughput of the system while maintaining the proportionality among all the users. Since all active users compete for the remaining power, the winner is determined according to the highest priority. Considering the constraint of rate proportionality, the priority is defined as,

$$\zeta_k = \hat{R}_k / \gamma_k \tag{9}$$

where $\widehat{R}_k$ denotes the bits that have been allocated to the kth user.

To maintain the constraint, the user who was allocated less bits is selected from the following minimization procedure

$$k' = \arg\min_{k} \zeta_k \tag{10}$$

Then one subcarrier is selected from the subcarriers, $\Omega_{k'}$, allocated to user k' to add one more bit. For user k', the incremental power due to adding one more bit to subcarrier n on which the allocated bit is less than b_{max} is given by

$$\begin{aligned} p_{k',n}^{(+)} &= \left(2^{\rho_{k',n} b_{k',n}+1} - 2^{\rho_{k',n} b_{k',n}}\right)\Gamma / H_{k',n} \\ &= 2^{\rho_{k',n} b_{k',n}}\Gamma / H_{k',n} \end{aligned} \tag{11}$$

which is obtained from (2) and (4). The subcarrier that requires the minimal incremental power is selected based on

$$n' = \arg\min_{n \in \Omega_{k'}} p_{k',n}^{(+)} \tag{12}$$

and then added with one bit

$$b_{k',n'} = b_{k',n'} + 1 \tag{13}$$

The procedure from (9) to (13) is repeated until all the remaining power is used up so that it is not enough to support any one more bit.

To be focused on the reallocation of the remaining power after the initial subcarriers-bit-power allocation, the proposed scheme adopts the existing resource allocation scheme (e.g. in [5]) with certain modifications as the initial allocation approach.

Then the proposed integer bit allocation scheme can be described as follows.

Step 1: Subcarrier allocation and Initial integer bits allocation

1) preliminary distribution of the number of subcarriers to each user

According to the rate proportionality, user k is assigned N_k subcarriers, that is,

$$N_k = \lfloor N/K \rfloor \gamma_k \tag{14}$$

Then the number of the possible unallocated subcarriers is,

$$N^* = N - \sum_{k=1}^{K} N_k \tag{15}$$

2) Setting $b_{k,n}$, $\rho_{k,n}$ and $\widehat{R}_k$ all to be zeros for all k and n; $\tilde{N} = \{1, 2, \cdots N\}$ is the index of the unallocated subcarriers. Assuming equal power allocation to all subcarriers, i.e,

$$p_{k,n} = \rho_{k,n} \frac{P_{total}}{N}$$

for all N subcarriers and all K users;

3) For each user k, the n'th subcarrier is selected based on

$$n' = \arg\max_{n \in \tilde{N}} |H_{k,n}|$$

to update the allocation for user k via function **UPDATING**(k, n'), which is described below.

```
4) while $\|\tilde{N}\| > N^*$,    % $\|\tilde{N}\|$ denotes the cardinality of the set $\tilde{N}$;
       selecting the user and the subcarrier
           $k' = \arg\min_{k} \zeta_k$,
           $n' = \arg\max_{n \in \tilde{N}} |H_{k',n}|$;
       if $N_{k'}>0$,    UPDATING($k', n'$);
       else $\zeta_{k'} = \infty$    % no available subcarrier for user $k'$
5) for $j$=1 to $N^*$
       $k' = \arg\min_{k} \zeta_k$,  $n' = \arg\max_{n \in \tilde{N}} |H_{k',n}|$,
       UPDATING($k', n'$);
       $H_{k',n'} = 0$    % no power will be allocated to user $k'$ on subcarrier $n'$
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Step 2: Power reallocation

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calculating $p_{k,n}^{(U)}$, $p_{k,n}^{(+)}$ and $P^{(A)}$,
while $P^{(A)}>0$
    $k' = \arg\min_{k} \zeta_k$,
    $n' = \arg\min_{n \in \Omega_{k'}} p_{k',n}^{(+)}$,
    if $b_{k',n'} < b_{max}$,
        $b_{k',n'} = b_{k',n'} + 1$,
        $\widehat{R}_{k'} = \widehat{R}_{k'} + b_{k',n'}$,
        $\zeta_k = \widehat{R}_k / \gamma_k$
        $p_{k',n'}^{(U)} = p_{k',n'}^{(U)} + p_{k',n'}^{(+)}$,
        $P^{(A)} = P^{(A)} - p_{k',n'}^{(+)}$
        $p_{k',n'}^{(+)} = 2^{b_{k',n'}}\Gamma / H_{k',n'}$,
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Function **UPDATING**(k,n) in the above scheme can be summarized as:

- determining integer bits $b_{k,n}$ according to (6)
- updating $\widehat{R}_k = \widehat{R}_k + b_{k,n}$,

  $\zeta_k = \widehat{R}_k / \gamma_k$,

  $\rho_{k,n'} = 1$,

  $N_k = N_k - 1$,

  $\tilde{N} = \tilde{N} - \{n\}$,

where '–' denotes set subtraction.

4. Simulation results

In this section, the performance of the proposed scheme is studied in comparison with that in [4] and [5] based on simulations.

The frequency selective multipath channel used in our simulations is the same as those in [4] and [5]. The channel is modeled as one consisting of 6 independent Rayleigh multipaths, with an exponentially decaying profile. A maximum delay spread of 5 μs and maximum Doppler of 30 Hz is assumed. The total bandwidth is 1MHz, and the total number of subcarriers is 64.

In the first case, 10 active users are assumed in the system. The variation of system throughput with different channel SNRs is calculated using the proposed scheme and is shown in Fig. 1. From the figure, it can be seen that reallocation of the remaining power leads to big improvement of the system throughput compared to the case without the reallocation of the remaining power. And the throughput from the proposed scheme is similar to that from Shen's method [4] and a little lower than that of Wong's method [5]. Note that the rounding-off is not taken into account in Shen's and Wong's methods in the simulations.

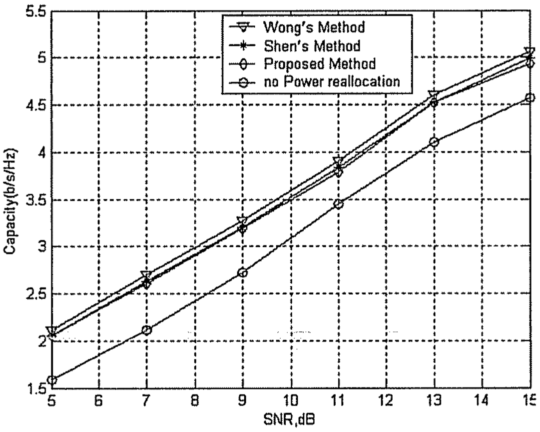

Fig. 1 Throughput versus SNR for 10 users

The variation of system throughput with the number of active users in the system is calculated using the proposed scheme and shown in Fig.2. Similar conclusions can be drawn from Fig. 2, that is, the proposed scheme has a similar performance to Shen's method [4] and a little inferior to that of Wong's method [5]. To compare with Shen's and Wong's methods under the condition of integer bits, these two methods are also implemented here with considering rounding-off, in terms of throughput. The rounding off scheme in [7] is used in the implementation. The calculated results are shown in Fig. 2 (named 'Integer Shen's Method' and 'Integer Wong's Method'). As shown in the figure, Shen's and Wong's methods' performance becomes worse after the rounding off, and so the proposed scheme is superior.

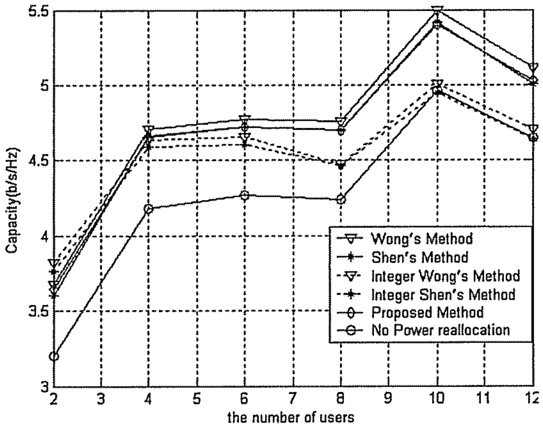

Fig. 2 Throughput versus users' number for SNR=15

The rate proportionality of 10 users for the three methods, compared to the proportionality constraint (given by γ_k), is depicted in Fig.3. Shen's method adheres to the proportionality constraint very well, while it can be observed that the proposed scheme can stick to the proportionality constraint better than Wong's method for some users.

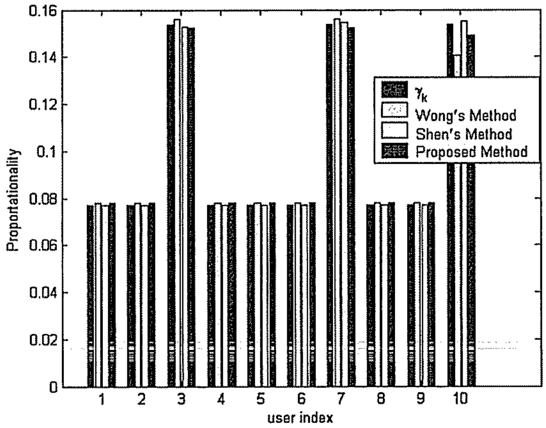

Fig.3 Variation of rate proportionality with user number

Fig.4 illustrates the used power in different resource allocation schemes. It can be noticed that there is quite a large amount of remaining power after the initial allocation in the proposed scheme (that is, before power reallocation). Reallocation of the remaining power results in the increase of throughput in the proposed scheme. The total power can be fully used up in Shen's and Wong's methods. In the proposed scheme in which integer bits allocation is made, there is only a very small amount of remaining power after resource reallocation.

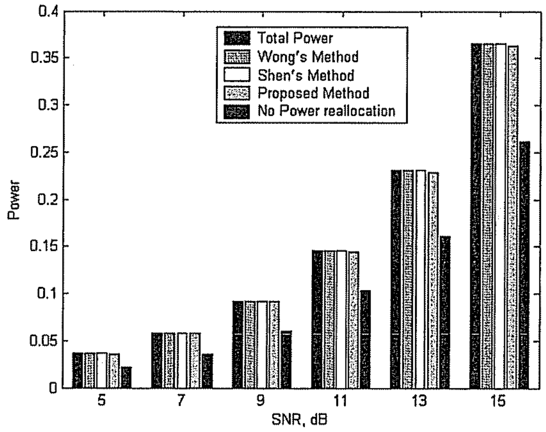

Fig.4 Variation of the used power with SNR

5. Conclusions

A subcarrier-power-integer bits allocation scheme, subject to constraints on both the total available power and the rate proportionality, is proposed to maximize the throughput of OFDMA systems. It conducts resource allocation in two steps: initial integer bits allocation, and power reallocation. By reallocating the remaining power due to integer bits allocation, the proposed scheme achieves the significant increase of throughput. Simulation is conducted and comparison of simulated results is made with those from two existing non-integer bits allocation schemes that, although the total available power can be can fully employed, does not include the operation of rounding off non-integer bits (the rounding off is necessary in the real situation!).

The comparison shows that the proposed scheme can perform similarly well to two non-integer bits allocation schemes without rounding off non-integer number, and can achieve better performance than the two non-integer schemes when the rounding off is taken into account.

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