Automatic Tuning of the Step size in WLMS Algorithms

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Introduction and summary

Estimation of time varying parameters in mobile radio communication is essential for high receiver performance.

The Wiener LMS (WLMS) algorithm is an efficient estimator for tracking of fading channels, in which prior statistical knowledge of the fading parameters can be incorporated in the algorithm design.

Fading channel parameters show trend behaviour and are therefore well described by Integrated Random Walks (IRW).

Optimal tuning of adaptive filters requires knowledge of the SNR and the fading rate, which are usually not known in advance. Therefore, we propose a variable step-size Wiener LMS (VSWLMS) scheme for adaptive tuning of the WLMS design parameters.

The Wiener LMS is here combined with variable step-size schemes designed for LMS. The variable step-size LMS is a special case of the VSWLMS obtained by assuming random walk (RW) parameters.

The proposed variable step-size Wiener LMS is here evaluated as a part of a receiver for the EDGE system. The performance of the VSWLMS is compared with fixed step-size schemes as well as with variable step-size LMS.

Summary: The proposed algorithm attains better tracking performance than both the variable step-size LMS (VSLMS) and the fixed step-size LMS and WLMS is severe fading conditions.

The Wiener LMS Algorithm

The Wiener LMS algorithm is a powerful tracking algorithm

Main features:

- Incorporates prior statistical knowledge about the channel
- Low complexity
- High performance
- Includes LMS as a special case

We consider tracking of the time-varying parameter vector h_t within the linear regression

$$y_t = \varphi_t^* h_t + v_t \qquad (1)$$

Transmission of symbols s_t over an M-tap time-dispersive channel yields

$$\varphi_t^* = (s_t, s_{t-1} \dots s_{t-M+1}) \qquad (2)$$

For white symbol sequences, $\mathbf{R} = E\varphi_t\varphi_t^* = \sigma_z^2 I$.

Basic Algorithm :

$$\varepsilon_t = y_t - \varphi_t^* \hat{h}_{t|t-1} \qquad (3)$$
$$\hat{h}_{t|t} = \hat{h}_{t|t-1} + \mu \mathbf{R}^{-1} \varphi_t \varepsilon_t \qquad (4)$$
$$\hat{h}_{t+k|t} = \mathcal{P}_k \hat{h}_{t|t} \qquad (5)$$

- $\hat{h}_{t+k|t}$: Estimate of h_{t+k}, based on data up to sample t
- μ : Step-size
- $\mathcal{P}_k$: IIR filter of the form

$$\mathcal{P}_k(z^{-1}) = \frac{Q_k(z^{-1})}{Q_0(z^{-1})}\mathbf{I} \qquad (6)$$

Design assumptions about the statistical characteristics of h_t provide a structured method for selecting Q_k.

Simplified WLMS design assumption: the tap-vector h_t is well modelled by the stochastic process

$$h_t + d_1 h_{t-1} + d_2 h_{t-2} = e_t \qquad (7)$$

The values of d_1 and d_2 are adjusted to capture the main characteristics of h_t. Assuming IRW behaviour corresponds to setting $d_1 = -2$ and $d_2 = 1$.

Under (7), the MSE optimal filter Q_k

$$Q_k(z^{-1}) = \mu(1 \;\; z^{-1})\begin{pmatrix} -d_1 & 1 \\ -d_2 & 0 \end{pmatrix}^k \begin{pmatrix} 1 \\ p \end{pmatrix} \qquad (8)$$

where

$$p = \frac{d_1 d_2(1-\mu)}{1+d_2(1-\mu)}$$

The LMS algorithm for white symbols is obtained with $d_1 = -1$ and $d_2 = 0$, i.e. random walk assumption, yielding $\mathcal{P}_k = 1$.

The optimal step-size μ depends on the SNR and the fading rate.

Summary simplified WLMS design:

Select d_1 and d_2, and adjust μ to minimize the MSE,

$$E|h_{t+k} - \hat{h}_{t+k|t}|^2 \qquad (9)$$

The Variable Step-size WLMS

The variable step-size WLMS is an evolution of the WLMS-algorithm that includes a gain adjustment algorithm originally developed for use in LMS.

Adjustment of the step-size μ on-line is here considered for scalar measurements and white symbol sequences, $\mathbf{R} = \sigma_z^2 I$.

By introducing $\bar{\mu} = \mu/\sigma_z^2$, the update of $\hat{h}_{t|t}$ in (4) can be expressed as

$$\hat{h}_{t|t} = \hat{h}_{t|t-1} + \bar{\mu}\varphi_t\varepsilon_t$$

We notice that $\bar{\mu}$ appears in the same way as the step-size in the LMS.

Hence, include in the WLMS structure a gain adjustment previously considered in the literature for use in variable step-size LMS (VSLMS)

$$\bar{\mu}_{t+1} = \bar{\mu}_t(1 + \rho\,\Re\{\psi_t\varphi_t\varepsilon_t\}) \qquad (10)$$
$$\psi_{t+1} = \psi_t(\mathbf{I} - \bar{\mu}_t\varphi_t\varphi_t^*) + \varphi_t^*\varepsilon_t^* \qquad (11)$$

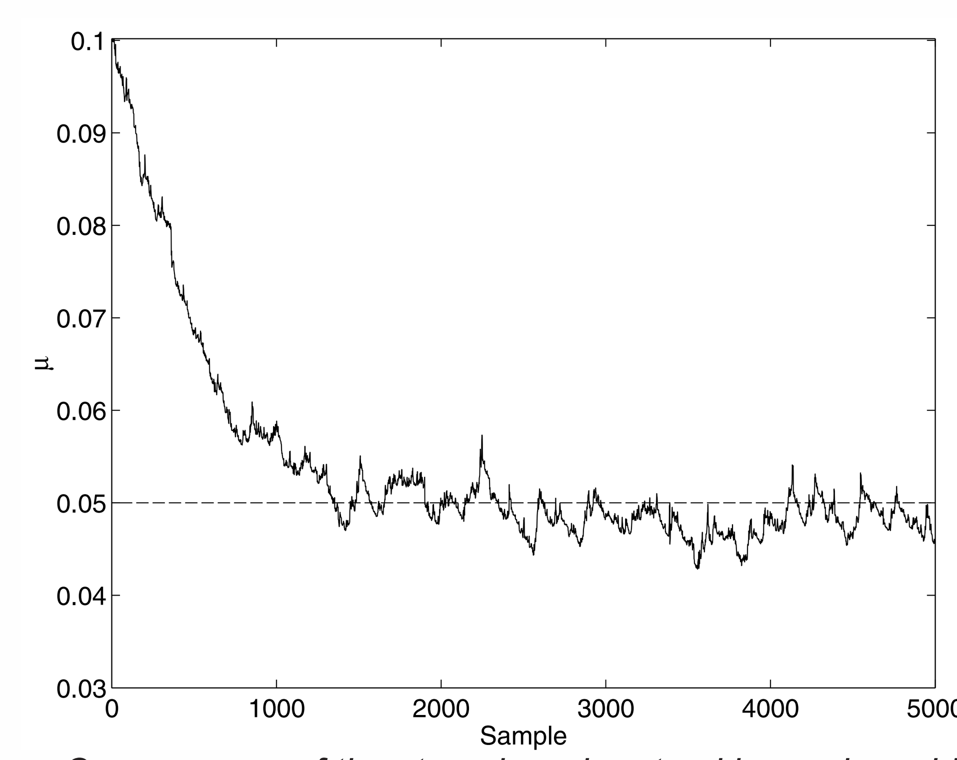
The variable step-size scheme (10),(11) is one of several possible schemes discussed in the literature that can be used with the WLMS.

ρ is a design parameter that balances convergence speed against noise sensitivity.

ψ_t associates with the derivative of the LMS filter weight with respect to the step-size parameter.

The variable step-size scheme (10),(11) combined with the simplified WLMS equations is

$$\varepsilon_t = y_t - \varphi_t^*\hat{h}_{t|t-1}$$
$$\hat{h}_{t|t} = \hat{h}_{t|t-1} + \bar{\mu}_t\varphi_t\varepsilon_t \qquad (12)$$
$$p_t = \frac{d_1 d_2(1-\bar{\mu}_t\sigma_z^2)}{1+d_2(1-\bar{\mu}_t\sigma_z^2)} \qquad (13)$$
$$\hat{h}_{t+1|t} = -p_t\hat{h}_{t|t-1} + (p_t - d_1)\hat{h}_{t|t} - d_2\hat{h}_{t-1|t-1}$$
$$\hat{h}_{t+k|t} = -p_t\hat{h}_{t+k-1|t-1} + Q_k\hat{h}_{t|t}\,, \quad k > 1\,.$$



Convergence of the step-size when tracking a sinusoid $e^{j0.005t}$ with the VSWLMS algorithm based on IRW $(d_1 = -2, d_2 = 1)$ and with ρ set to 0.08. Optimal step-size (dashed) reached after approximately 1000 iterations.

Application to EDGE

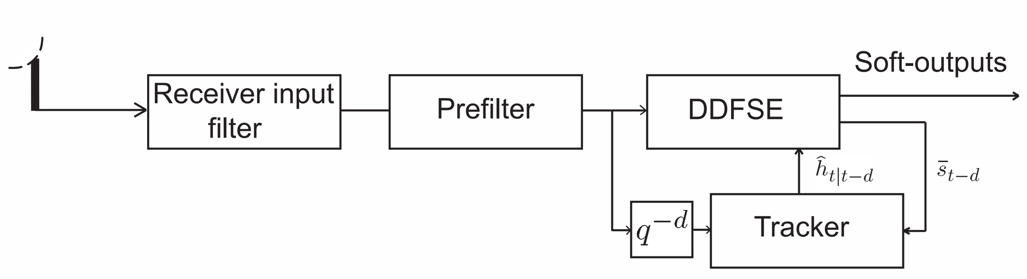
The block error rate (BLER) performance of a delayed decision-feedback sequence equalizer (DDFSE) in conjunction with the VSWLMS algorithm is now evaluated on a fading channel associated with the EDGE air interface.

The EDGE slot format

156.25 symbols (0.577 ms)

3 tail	58 data	26 sync	58 data	3 tail	8.25 guard

The EDGE receiver model



Since the tracker works on d-step delayed data, d-step prediction has to be performed.

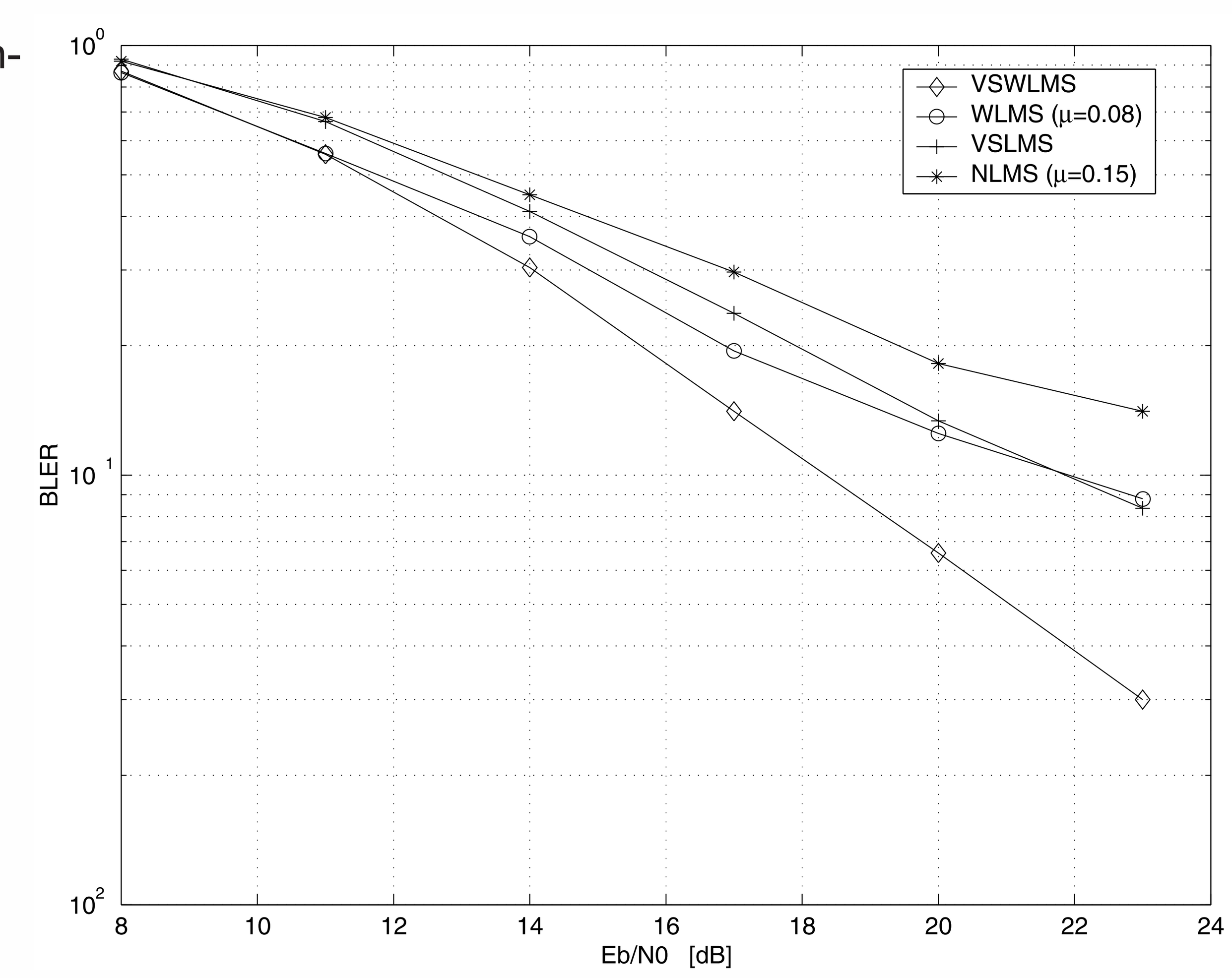


Figure 1. Block error rate (BLER) when transmitting over a rather severe fading channel at the 1800 MHz band. (Prefilter: 15-taps, decision delay: 3 symbols, WLMS design: IRW). Ideal frequency hopping is assumed. For both VSWLMS and the VSLMS, the design parameter ρ is set to 0.08 and the variable step-size $\bar{\mu}_t$ is adapted over the slots. Mobile speed: 200 km/h. Frequency offset: 200Hz.

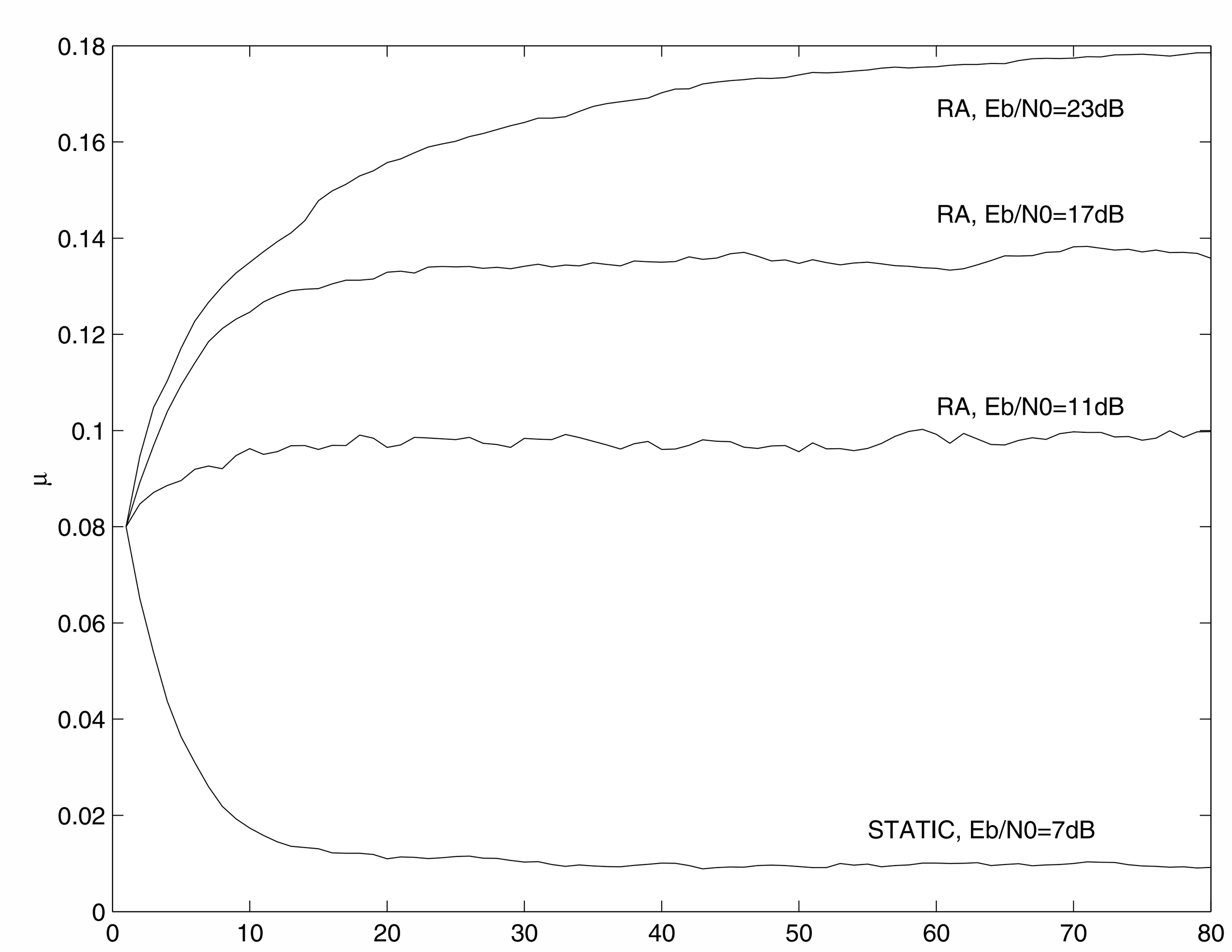


Figure 2. The ensamble mean of 50 realizations of the adaptively tuned step-size μ versus the slot number. ρ = 0.08.