Table of formulas

Sums involving geometric series

$$\sum_{k=n_1}^{n_2} x^k = \frac{x^{n_2+1} - x^{n_1}}{x-1} = \frac{x^{n_1} - x^{n_2+1}}{1-x} \tag{1}$$

$$\sum_{k=n_1}^{n_2} kx^k = \frac{x^{n_2+1}((n_2+1)(x-1)-x)}{(x-1)^2} - \frac{x^{n_1}(n_1(x-1)-x)}{(x-1)^2} = \frac{(n_2+1)x^{n_2+1} - n_1 x^{n_1}}{x-1} - \frac{x^{n_2+2} - x^{n_1+1}}{(x-1)^2} \tag{2}$$

$$\sum_{k=n_1}^{n_2} k^2 x^k = \frac{(n_2+1)^2 x^{n_2+1} - n_1^2 x^{n_1}}{x-1} - 2x\frac{(n_2+1)x^{n_2+1} - n_1 x^{n_1}}{(x-1)^2} + \frac{(x^2+x)(x^{n_2+1} - x^{n_1})}{(x-1)^3} \tag{3}$$

Properties of binomial coefficients

1. Definition:
$$\binom{R}{r} = \frac{R!}{(R-r)!r!} \tag{4}$$

2.
$$\frac{\binom{R+n}{r+n}}{\binom{R}{r}} = \frac{(R+n)(R+n-1)\dots(R+1)}{(r+n)(r+n-1)\dots(r+1)} \tag{5}$$

Sums involving binomial coefficients

1.
$$\sum_{k=0}^{K} \binom{K}{k} = 2^K \tag{6}$$

2.
$$\sum_{k=0}^{K} \binom{K}{k} f^k = (1+f)^K \tag{7}$$

3.
$$\sum_{r=0}^{k} \binom{n}{r} f^r (1-f)^{n-r} = \left(f - 1 + f^{k+1}(1-f)^{n-k}\binom{n}{k+1}\right) \tag{8}$$

4.
$$\sum_{k=0}^{K} \binom{K}{k}^2 = \frac{2K!}{(k!)^2} \tag{9}$$

5.
$$\sum_{R=0}^{N} \binom{R}{r}\binom{N-R}{n-r} = \binom{N+1}{n+1} \tag{10}$$

The beta integral

1.
$$\int_0^1 \theta^r (1-\theta)^{n-r} d\theta = \beta(r+1, n-r+1) \tag{11}$$
where $\beta(z, w)$ is the beta-function (beta in Matlab).

2. If n and r are integers and $r \leq n$,
$$\beta(r+1, n-r+1) = \frac{r!(n-r)!}{(n+1)!} \tag{12}$$

3.
$$\int_a^b \theta^r (1-\theta)^{n-r} d\theta = \beta(r+1, n-r+1) \left[I_b(r+1, n-r+1) - I_a(r+1, n-r+1) \right] \tag{13}$$
where $I_a(z, w)$ is the incomplete beta function (betainc in Matlab).