Exercises MCMC

Numerical methods for model prediction. Consider the model in eq. (3) in "Bayesianska numeriska metoder I". Your task is to predict unseen y-values from this model for $x \in [0, 2]$, given a set of measurements $\mathcal{X} = \{x_1, ..., x_N\}$ and $\mathcal{Y} = \{y_1, ..., y_N\}$ and the priors given in the text. Present your results as the mean $\pm$ std. Note that the standard deviation will be depend on x. Use as the true parameter values $w_0 = 1.5$ and $w_1 = 0.5$.

Consider the following four data sets

1. $N = 1$, $x_1 = 1$ and $y_1 = 0.5$.
2. $N = 1$, $x_1 = 2$ and $y_1 = \sqrt{2}$.
3. $N = 2$, use both the two above data points
4. $N = 10$, generate ten random points from the true model.

   Choose one of the methods below to solve the task:

   - Analytical integration using the normal approximation. Present your derivations.
   - Metropolis-Hastings. Use a circular proposal density with step lenght, ϵ. Choose ϵ so that the acceptance rate is above 0.5. Start your simulations from a point well outside the bulk of the posterior distribution and find a practical method to judge when the algorithm has reached a steady state (when the "burn in" is over). Plot the trajectory of the samples for at least one run.
   - Gibb's sampling. Explain how you solved the problem of sampling from the conditional distributions. Start your simulation from a point well outside the bulk of the posterior distribution and find a practical method to judge when the algorithm has reached a steady state (when the "burn in" is over). Plot the trajectory of the samples for at least one run.

   Write down your comments concerning what you believe to be the pros and cons of the method you chose. It will be interesting to compare our results.