Exercises chapter 3

1. **Reasoning back and forth.** Exercise 3.6 in the book. Note that it might be difficult to make a neat mathematical analysis that help you answer the questions (a)-(c) and also that the questions asked are related to exercise 2 where you are supposed to implement the formulas in matlab (for instance). Thus, use numerical examination instead of theoretical analysis if that helps you. Play around with the numbers N, M, ϵ, and δ to get some intuition and make plots for illustrative cases.

2. **Reasoning back and forth, cont'd.** Implement the formulas for $P(R_j|R_k, C)$ and $P(R_k|R_j, C)$ with $j < k$ for the case considered in section 3.9. Then, for some suitable values of N, M, and n, plot $P(R_j|R_k, C)$ as a function of j (with a fixed k) and plot $P(R_k|R_j, C)$ as a function of k (with fixed j). Examine/verify numerically the following properties:

   (a) The symmetry of forward and backward inferences for the case when $p\epsilon = q\delta$.

   (b) The probabilities $P(R_j|R_k, C)$ for small values of j when $p\epsilon \neq q\delta$. Explain the behavior. In particular, examine and explain the result for the case $P(R_1|R_k, C)$ for k being relatively large. (Compare with $P(R_k|R_1, C)$ for the same k.)

3. Implement the formula for $P(R_k|R_{\text{later}}, B)$ in eq. (3.56). Plot, for fixed N, M, and n, $P(R_k|R_{\text{later}}, B)$ as a function of k. Choose N, M, and n, in the way that various aspects of the results are simple to point out. For instance, can you find any special cases for which $P(R_k|R_{\text{later}}, B)$ also can be derived in a simple way, without the need for the relatively complicated formula in eq. (3.56)?