Plausible Reasoning

Chapter 1 in Jaynes' Probability Theory

Annea Barkefors

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Outline

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The Origin

- We want to have a mathematical theory, that is able to reason in a fashion similar to the way we do
- With this in mind, we imagine a robot, with a brain capable of plausible reasoning

Deductive Reasoning

Deductive reasoning makes use of two strong syllogisms, from which conclusions can be reached

if A is true, then B is true	if A is true, then B is true
A is true	B is false
therefore, B is true	therefore, A is false

Plausible Reasoning

Plausible reasoning use weaker syllogisms, from which no definite conclusions can be reached, but we become more or less certain of our proposition

if A is true, then B is true

B is true

---

therefore, A is more plausible

Example:

$A \equiv$ we will play volleyball at 15
$B \equiv$ the sky will be sunny before 15

Plausible Reasoning

if A is true, then B is true

A is false

---

therefore, B becomes less plausible

if A is true, then B becomes more plausible

B is true

---

therefore, A becomes more plausible

Conditional Plausibility

The plausibility that the robot assigns to some proposition A will, in general, depend on whether we told it that some other proposition B is true.

This is called *conditional plausibility*, and is indicated by $A|B$

Boolean Algebra

- Boolean algebra is the algebra of logic. We have propositions $A, B, \ldots,$ that can either be *true* or *false*
- The *equations* consists of assertions that the propositions on the left-hand side is logically equivalent to the one on the right-hand side
- Two propositions A and B have the same *truth value* if one is true if and only if the other is true: they are logically equivalent propositions

Operations

Boolean algebra has several operations:

- logical product / conjunction: AB
- logical sum / disjunction: $A + B$
- denial: $\overline{A} \equiv A$ is false
- implication: $A \implies B \quad (A = AB)$

- NAND: $A \uparrow B \equiv \overline{AB} = \overline{A} + \overline{B}$
- NOR: $A \downarrow B \equiv \overline{A + B} = \overline{A}\,\overline{B}$

With the help of these operations, any logical function can be generated.

The Basic Desiderata

Desideratum I

Degrees of plausibility are represented by real numbers.

Desideratum II

Qualitative correspondence with common sense

Desideratum III

a) If a conclusion can be reasoned out in more than one way, then every possible way must lead to the same result.
b) The robot always takes into account all of the evidence it has relevant to a question.
c) The robot always represents equivalent states of knowledge by equivalent plausibility assignments.

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The pink's book approach to probability uses axiomatic definitions:

$$\text{axiom 1: } P(A) \geq 0$$

$$\text{axiom 2: } P(S) = 1$$

$$\text{axiom 3: } P\left(\bigcup_{n=1}^{N} A_n\right) = \sum_{n}^{N} P(A_n) \quad \text{if } A_m \cap A_n = \varnothing$$

What does Jaynes think about set theory and Venn diagram?

The points in the circle A "must represent some ultimate 'elementary' propositions ω_i into which A can be resolved. ... But the general theory we are developing has no such structure; ... these things are properties only of the Venn diagram."

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The *conditional probability* of an event A, given B, is defined as:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

The events A and B are *statistically independent* if:

$$P(A|B) = P(A), \quad P(B|A) = P(B)$$

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"The plausibility that the robot assigns to some proposition A will, in general, depend on whether we told it that some other proposition B is true"
"We may call [this] 'the conditional probability that A is true, given that B is true'"

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